

Exponential & Radical Relationships Homework

Directions: Be sure to show all work, communicate your thought process, and justify your reasoning. Remember to check that your answers are complete, correct, and reasonable. Do not forget to complete the “Throwback” problems! ☺

Section A: Write each expression in exponential form. Simplify as much as possible. (no radicals in final answer, but you may have fractional exponents in final answer)

1. $\sqrt{50r^9s^{13}t^{20}}$

2. $\sqrt[4]{81^3}$

3. $\sqrt[3]{125^4}$

4. $\sqrt[5]{-243t^{13}u^{25}v^{18}}$

5. $\sqrt{121s}$

6. $\sqrt[6]{64x^4}$

7. $(\sqrt[7]{q^{63}r^{7}s^{14}t})$

8. $\sqrt[8]{(1-q)^3}$

9. $\sqrt[3]{-27h^9j^{40}}$

10. $(\sqrt[3]{27p^{12}q^{15}})^4$

11. $\sqrt[3]{\sqrt{(w+2)^{-3}}}$

12. $(\sqrt[4]{16v^{24}q^{80}})^2$

13. **Multiple Choice:** Which expression is equivalent to $(\sqrt[3]{243})^4$?

- a. 20 b. 3^5 c. 9^2 d. 81^4

14. **Multiple Choice:** Which expression is equivalent to $(\sqrt[2]{128})^8$?

- a. 56 b. 2^7 c. 16^4 d. 4^4

Section B: Write each expression using radicals and/or integer exponents. Simplify as much as possible. (no fractional exponents in final answer)

15. $\left(\frac{81r^2s^3}{r^8s^{\frac{1}{3}}}\right)^{-\frac{1}{4}}$

16. $81^{\frac{3}{4}} * 8^{\frac{2}{3}}$

17. $\frac{(x^8y^5)^{\frac{1}{2}}}{(xy)^{-1}}$

18. $(-125v^6w^{15}x^4)^{\frac{2}{3}}$

19. $\frac{(9z)^{\frac{4}{3}}}{(9z)^{\frac{10}{3}}}$

20. $(16k^{12}m^7n^4)^{\frac{3}{4}}$

continued on next page... (solutions are on the last pages!)

21. $\left(\frac{r^3 s^{\frac{3}{2}}}{r^{\frac{2}{3}} s}\right)^{-2}$

22. $(u^8 q^2 w^3)^{\frac{3}{8}}$

23. $32^{\frac{3}{5}} * -8^{\frac{4}{3}}$

24. $128^{\frac{3}{7}}$

25. $(-125y^7)^{\frac{2}{3}}$

26. $64^{-\frac{3}{2}}$

27. $\frac{(3x)^3}{(3x)^{\frac{7}{2}}}$

28. $\frac{7(\sqrt[4]{s^3})}{\sqrt[3]{s^4}}$

29. $343^{-\frac{2}{3}}$

 THROWBACK!

30. The expression $x^4 - 256$ can be rewritten in the form $(x^2 + a)(x + b)(x + c)$.
What are the values of a, b, and c?

31. Write the correct factored form for the function having zeros $\frac{3}{4}$ (M. 2), -1, and 4(M.3).

32. Factor $3x^4 - 11x^2 - 4$.

33. How many solutions will the equation $-x - 1 = |x| + 5$ have?
(Hint: Think in terms of the number of intersections of the graphs of the functions!)

Solutions: see next pages!

DO NOT forget to complete the "Throwback" problems! ☺

Section A: Write each expression in exponential form. Simplify as much as possible.

$$1. \sqrt{50r^9s^{13}t^{20}}$$

$$= \sqrt{25 \cdot 2 \cdot r^9 \cdot s^{13} \cdot t^{20}}$$

$$= 5r^4s^6t^{10}\sqrt{2rs}^{1/2}$$

$$4. \sqrt[3]{-243t^{15}u^{25}v^{18}}$$

$$= \sqrt[3]{-243t^{15}u^{15}v^{15} \cdot 3 \cdot 5 \cdot 5 \cdot 5 \cdot 3 \cdot 3 \cdot 3}$$

$$= (-3t^5u^5v^3(t^3v^3)^{1/5})$$

$$7. \sqrt[3]{q^6r^7s^4t^7}$$

$$= q^2rs^2t^7$$

$$10. \sqrt[4]{27p^{12}q^{15}}$$

$$= (3p^3q^5)^{4/4}$$

$$= 3p^3q^5$$

$$2. \sqrt[3]{81^3}$$

$$= (81^3)^{1/3}$$

$$= \sqrt[3]{81^3} = 81$$

$$5. \sqrt{121s}$$

$$= 11\sqrt{s}$$

$$= (11s^{1/2})$$

$$3. \sqrt[4]{125^4}$$

$$= (125^4)^{1/4}$$

$$= \sqrt[4]{125^4} = 125$$

$$6. \sqrt[4]{64x^4}$$

$$= 2x$$

$$= (2x^1)$$

$$9. \sqrt[3]{-27h^9j^{30}}$$

$$= \sqrt[3]{-27h^9j^{30}}$$

$$= -3h^3j^{10}$$

$$12. \sqrt[2]{16w^{24}q^{80}}$$

$$= (2\sqrt[2]{16w^{24}q^{80}})^2$$

$$= 4w^{12}q^{40}$$

$$11. \sqrt[3]{(w+2)^{-3/2}}$$

$$= \frac{1}{\sqrt[3]{(w+2)^{3/2}}}$$

$$d. 81^4$$

$$c. 9^2 = 81$$

$$b. 3^5 = 243$$

$$a. 20$$

$$13. \sqrt[4]{243^4}$$

$$= (243^4)^{1/4} = 243$$

$$14. \sqrt[8]{128^8}$$

$$= 128$$

$$c. 16^4$$

$$b. 2^7$$

$$a. 56$$

$$d. 4^4$$

Section B: Write each expression using radicals. Simplify as much as possible.

$$3^{-\frac{1}{2}} = \frac{1}{\sqrt{3}}$$

$$15. \left(\frac{81r^2s^3}{r^2s^3}\right)^{-1/4} = (81r^{-6}s^3)^{-1/4} = 81^{-1/4} r^{6/4} s^{-3/4} = \sqrt[4]{81} r^{3/2} s^{-3/4} = \frac{3\sqrt[4]{81}}{3\sqrt[4]{81}s^{3/4}} = \frac{1}{\sqrt[4]{81}s^{3/4}}$$

$$16. 81^{1/4} \cdot 8^{3/4} = (81 \cdot 8)^{3/4} = 4 \sqrt[4]{81 \cdot 8} = 4 \sqrt[4]{648} = 4 \sqrt[4]{2^3 \cdot 3^4} = 4 \cdot 3 \sqrt[4]{2^3} = 12 \sqrt[4]{8}$$

$$17. \frac{(x^2y)^4}{(xy)^4} = x^4 y^4 \cdot \frac{1}{x^4 y^4} = x^4 y^4 \cdot x^{-4} y^{-4} = x^0 y^0 = 1$$

$$18. (-125)^{2/3} = (\sqrt[3]{-125})^2 = (-5)^2 = 25$$

$$19. \frac{(9z)^{1/3}}{(9z)^{2/3}} = (9z)^{1/3 - 2/3} = (9z)^{-1/3} = \frac{1}{\sqrt[3]{9z}}$$

$$20. (16k^3m^2n^3)^{1/4} = \sqrt[4]{16k^3m^2n^3} = 2 \sqrt[4]{k^3m^2n^3}$$

$$21. \left(\frac{r^3s^2}{r^2s}\right)^{-2} = \left(\frac{r^1s^1}{r^2s}\right)^{-2} = \left(\frac{r}{s}\right)^{-2} = \frac{1}{\left(\frac{r}{s}\right)^2} = \frac{s^2}{r^2}$$

$$22. (4x^2y^3)^{3/4} = \sqrt[4]{(4x^2y^3)^3} = \sqrt[4]{64x^6y^9} = 2 \sqrt[4]{4x^6y^9} = 2 \sqrt[4]{2^2x^6y^9} = 2 \sqrt[4]{2^2} \sqrt[4]{x^6y^9} = 2 \sqrt[4]{2} \sqrt[4]{x^6y^9}$$

$$23. 32^{3/5} = (\sqrt[5]{32})^3 = (2)^3 = 8$$

$$24. \sqrt[3]{128} = \sqrt[3]{2^7} = 2 \sqrt[3]{2^4} = 2 \sqrt[3]{16}$$

$$25. (-125)^{2/3} = (\sqrt[3]{-125})^2 = (-5)^2 = 25$$

$$26. 64^{3/4} = (\sqrt[4]{64})^3 = (2)^3 = 8$$

$$27. \frac{(3x)^3}{(3x)^2} = 3x$$

$$28. \frac{3}{4} - \frac{1}{3} = \frac{9}{12} - \frac{4}{12} = \frac{5}{12}$$

$$29. \sqrt[3]{343} = \sqrt[3]{7^3} = 7$$

$$30. \sqrt[4]{16} = 2$$

$$31. \sqrt[4]{81} = 3$$

$$32. \sqrt[3]{27} = 3$$

$$33. \sqrt[4]{16} = 2$$

$$34. \sqrt[3]{27} = 3$$

Section B: Write each expression using radicals. Simplify as much as possible.

15. $(\frac{81r^3s^3}{r^3s^3})^{-1/4}$
 $= (81r^3s^3)^{-1/4}$
 $= 81^{-1/4} r^{3 \cdot -1/4} s^{3 \cdot -1/4}$
 $= 3^{-1/4} r^{-3/4} s^{-3/4}$
 $= \frac{1}{\sqrt[4]{3^3 r^3 s^3}}$

16. $\frac{(x^2y^3)^{1/2}}{(xy)^{-1}}$
 $= x^{1/2} y^{3/2} \cdot xy$
 $= x^{5/2} y^{5/2}$
 $= \sqrt[5]{x^5 y^5}$

17. $\frac{(x^2y^3)^{1/2}}{(xy)^{-1}}$
 $= x^{1/2} y^{3/2} \cdot xy$
 $= x^{5/2} y^{5/2}$
 $= \sqrt[5]{x^5 y^5}$

18. $(-125w^{15}x^{15})^{2/3}$
 $= (-125)^{2/3} w^{15 \cdot 2/3} x^{15 \cdot 2/3}$
 $= (-5)^2 w^{10} x^{10}$
 $= 25w^{10}x^{10}$

19. $(\frac{9z^2}{9z^2})^{1/3}$
 $= (1)^{1/3}$
 $= 1$

20. $(16x^3m^2n^4)^{3/4}$
 $= (2^4)^{3/4} x^{3 \cdot 3/4} m^{2 \cdot 3/4} n^{4 \cdot 3/4}$
 $= 2^3 x^{9/4} m^{3/2} n^3$
 $= 8x^{2.25} m^{1.5} n^3$

21. $(\frac{r^{3/2}}{r^{1/5}})^{-2}$
 $= (r^{3/2 - 1/5})^{-2}$
 $= (r^{13/10})^{-2}$
 $= r^{-26/10}$
 $= r^{-13/5}$
 $= \frac{1}{r^{13/5}}$

22. $(4r^2w^3)^{1/4} (8w^6)^{3/4}$
 $= (4^2)^{1/4} r^{2 \cdot 1/4} w^{3 \cdot 1/4} \cdot (8^3)^{1/4} w^{6 \cdot 3/4}$
 $= 2 r^{1/2} w^{3/4} \cdot 2^3 w^{9/2}$
 $= 2^4 r^{1/2} w^{3/4 + 9/2}$
 $= 16 r^{1/2} w^{21/4}$

23. $32^{2/3} \cdot 8^{1/3}$
 $= (2^5)^{2/3} \cdot 2^1$
 $= 2^{10/3} \cdot 2^1$
 $= 2^{13/3}$
 $= 2^4 \cdot 2^{1/3}$
 $= 16 \sqrt[3]{2}$

24. $\frac{(3x)^3}{(3x)^2}$
 $= 3x$

25. $(-125y^3)^{2/3}$
 $= (-125)^{2/3} y^{3 \cdot 2/3}$
 $= (-5)^2 y^2$
 $= 25y^2$

26. $64^{3/2}$
 $= (2^6)^{3/2}$
 $= 2^{9}$
 $= 512$

27. $(3x)^{-1/2}$
 $= \frac{1}{\sqrt{3x}}$

28. $\frac{7\sqrt[3]{s}}{\sqrt[3]{s^5}}$
 $= \frac{7s^{1/3}}{s^{5/3}}$
 $= 7s^{-4/3}$
 $= \frac{7}{s^{4/3}}$
 $= \frac{7}{\sqrt[3]{s^4}}$

29. $343^{2/3}$
 $= (7^3)^{2/3}$
 $= 7^2$
 $= 49$

Throwback:

$$1. x^4 - 256 = (x^2 + 16)(x^2 - 16) = (x^2 + 16)(x + 4)(x - 4).$$

Therefore, a = 16, b = 4, and c = -4.

$$2. (4x - 3)(4x - 3)(x + 1)(x - 4)(x - 4)(x - 4)$$

$$3. (3x^2 + 1)(x^2 - 4) = (3x^2 + 1)(x - 2)(x + 2)$$

4. The equation will have no solutions. The slope of $-x - 1$ is the same as the slope of the left side of the graph of $|x|$, making them parallel. $-x - 1$ is a line that will pass through (0,-1) and the vertex of the absolute value function $|x| + 5$ is (0,5). Therefore, the graphs will never intersect, implying that no values for x will result in the same output for both functions.