

**Exponential Functions Homework**

**Directions:** Be sure to show all work, communicate your thought process, and justify your reasoning. Remember to check that your answers are complete, correct, and reasonable. Do not forget to complete the “Throwback” problems! ☺

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**1. An isotope of Plutonium, Plutonium-240 has a half-life of 6,563 years (meaning its mass decreases by half every 6,563 years). Plutonium-240 can be used to analyze samples that are under 500,000,000 years old.**

- Write a function  $M(t)$  to represent the mass of Plutonium-240 in a sample that is  $t$  years old. Assume the original mass is  $x$  grams. Represent the function once using a positive exponent, and once using a negative exponent.
- What is the domain of  $M(t)$ ?
- Is  $M(t)$  linear or nonlinear? Is  $M(t)$  increasing or decreasing?
- After how many years will the amount of Plutonium-240 be one-eighth of the original amount?

**2. The population of a certain town is expected to increase by 2.4% every three years.**

- If there are currently 17,238 people in the town, what would you expect the population to have been three years ago?
- Write a function  $P(t)$  that can be used to estimate the population,  $P$ , of the town  $t$  years from now.
- Determine the average rate of change in the expected population from now until 2022, as well as the average rate of change in the expected population from 2022 to 2037. Is the average rate of change in the expected population increasing or decreasing over time?
- A student claims that since the population is expected to increase by 2.4% every three years, we can expect the population to increase by a total of 7.2% the first nine years. Do you agree or disagree? Explain.

**3. Francis has a statue that was valued at \$525 in 2008, and appraised to increase in value by 1.8% annually.**

- Write a function  $V(x)$  to represent the estimated value of the statue  $x$  years after 2008.
- How much would you expect the statue to be worth now?
- Determine the average rate of change in the expected value of the statue from 2008 until now, as well as the average rate of change in the expected value of the statue from now until 2020. Is the rate of change increasing or decreasing over time?
- When can Francis expect to receive over \$800 for the statue? Explain.

4. Kyle borrowed \$175 from his sister to pay for bicycle repairs. He agrees to pay his sister \$80 today, and half of what he paid the previous week each week. His sister does not agree to this deal! Why?

5. A company donates annually to a scholarship foundation. Each year, they increase the amount of the donation by 12%. The total amount of money the company donated to the foundation is \$1542.05 after five years.

a. Determine the initial donation, to the nearest dollar.

b. If the company had rather donated \$1000 to start, but increased by 8% per year, would their yearly average donation be higher or lower over the first 10 years? Thoroughly support your answer.

c. *Creation challenge!* After 10 years, the company decides to change their plan, but would still like to have the same average annual donation (give or take one dollar) per year for the next 10 years. Choose any initial donation amount and rate per year that would result in the same average yearly donation over the first 10 years as the original situation.

### THROWBACK!

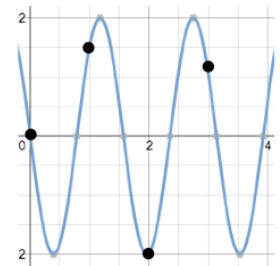
The graph models the height,  $h$ , above the ground in feet at  $t$  seconds of a ball bouncing. Order the time intervals from least to greatest, according to the average rate of change.

Interval A: from 0 seconds to 1 second

Interval B: from 1 second to 2 seconds

Interval C: from 2 seconds to 3 seconds

Least to greatest: Interval \_\_\_\_\_, Interval \_\_\_\_\_, Interval \_\_\_\_\_



**Selected Solutions:**

1. a.  $M(t) = x\left(\frac{1}{2}\right)^{t/6563}$  and  $M(t) = x(2)^{-t/6563}$

b. [0, 500000000)

c. nonlinear; decreasing

d. 19,689 years

3. a.  $V(x) = 525(1.018)^x$

b.  $x = 8$ ;  $V(x) = \$605.54$

c.

x	V(x)
2008: x = 0	525
Now: x = 8	605.54

x	V(x)
Now: x = 8	605.54
2020: x = 12	650.33

Average rate of change: \$10.07  
per year

Average rate of change: \$11.20  
per year

The average rate of change is increasing over time.

5. a. \$875

b. When donating an initial amount of \$875, the total donation after ten years will be  $875(1.12)^{10}$ , or \$2717.62. This results in an average of \$271.76 per year.

When donating an initial amount of \$1000, but decreasing the rate to 8%, the total donation after ten years will be  $1000(1.08)^{10}$ , or \$2158.92. This results in an average of \$215.89 per year. Thus, the average yearly donation would be lower.

c. will vary

example: initial donation of \$900, rate of 11.68%

**Throwback:**

B, A, C (remember, rate of change is also known as "slope!")