Root Theorems Class Work

✦ **Objective:** *You will be able to identify roots of polynomial expressions, and write polynomial expressions provided with their roots.*

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✎ **Quick Review:**

— zero, solution, etc.

– What is a root of a polynomial?

find the factors ⇒ set equal to zero (zeroes) and solve! *and*

– How can you determine if a factor is a root, using polynomial division?

★ **Rational Root Theorem:**

The possible real <u>rational</u> roots of any polynomial are the ratios of the factors of the constant term and factors of the <u>leading</u> coefficient!   *fractional*   *first*

Guided Example A: Determine all roots of the function
$$3x^3 + x^2 - 4x + 12 = 0.$$

cubic 3 roots

$$\underline{\text{Possibilities}}$$

$$\frac{\text{factors of constant}^{(12)}}{\text{leading coefficient}(3)} = \frac{\pm 1, \mp 2, \pm 3, \pm 4, \pm 6, \pm 12}{\pm 1, \pm 3}$$

$$\text{possible} = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$$
$$\pm 1/3, \pm 2/3, \pm 4/3$$

Check:

$x=1$

3	1	-4	12
	3	4	0
3	4	0	12

not a root b/c remainder!

$x=-2$

3	1	-4	12
	-6	10	-12
3	-5	6	0

$$3x^2 - 5x + 6 = 0$$
$$x = \frac{5 \pm \sqrt{25-72}}{6}$$
$$x = \frac{5 \pm \sqrt{-47}}{6} = \frac{5 \pm i\sqrt{47}}{6}$$

$$x = -2, \frac{5+i\sqrt{47}}{6}, \frac{5-i\sqrt{47}}{6}$$

Guided Example B: Determine all roots of the function
$$2x^4 - 9x^3 + 22x^2 - 81x + 36 = 0.$$

Possible: factors of $\dfrac{36}{2}$

$$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36$$
$$\pm 1/2, \pm 3/2, \pm 9/2$$

Trial & Error: Tried 1, 2, 3

$x=4$

2	-9	22	-81	36
	8	-4	72	-36
2	-1	18	-9	0

$$2x^3 - x^2 + 18x - 9 = 0$$ *4 terms ⇒ group!*
$$x^2(2x-1) + 9(2x-1)$$
$$(2x-1)(x^2+9) = 0$$

$$2x - 1 = 0 \qquad x^2 + 9 = 0$$
$$\boxed{x = 1/2} \qquad \sqrt{x^2} = \sqrt{-9}$$
$$\boxed{x = \pm 3i}$$

$$x = 4, 1/2, 3i, -3i$$

✎ **Practice:** Determine all roots of each function,
as well as their multiplicities.

1. $3x^4 + 7x^3 - 73x^2 - 175x - 50 = 0$

HW problem for 02.28 night

2. $4x^3 - 14x^2 + 13x - 21 = 0$