Name: ______________________________ Date: ______________________ **Unit 7 Class Work**

Solving Exponential Equations (Applied Problems) Class Work

Objective: *You will be able to solve problems involving exponential situations.*

★ **Quick Review:** Solve each equation for the variable.

A. $2 = 4e^{-x/10}$

B. $\ln(3x - 2) = 2$

C. $300 = 2000^{-.005x}$

D. $2^{x+4} = 3^{x-1}$

E. $10 * 3^{x/3} = 80$

★ **Example 1:** A radioactive substance decays at a rate of 3.5% per hour. Consider a sample of 250 mg of a radioactive substance.

a. Write an exponential equation to represent A, the amount of the substance, as a function of t, the number of minutes the sample has been observed.

b. How many minutes, to the nearest tenth of a minute, will it take for the substance to decay to less than 10 mg?

Practice: Solve each problem! Support your answers. Be sure to check for reasonableness also.

1. A scientist has a sample of bacteria that initially contains 20 million microbes. She determines that the number of bacteria triples every half an hour!

a. Write an exponential equation to represent N, the total number of microbes (in millions) as a function of x, the number of minutes the sample has been observed.

b. Approximately how many minutes will it take for there to be 200 million microbes?

★ **Example 2:** A certain radioactive isotope decays exponentially according to the model $A = A_o e^{-kt}$, where k is the rate of decay, A is the number of grams of the isotope at the end of t days, and A_0 is the initial mass of the isotope in grams. This isotope decays at a rate of 13% per day. Determine the half-life of the isotope.

✐ **<u>Practice</u>:** Solve each problem! Support your answers. Be sure to check for reasonableness also.

2. The formula $P = P_0 * 2^{-h/4795}$ can be used to determine the atmospheric pressure, P, in millimeters of mercury, at an elevation of h meters above sea level. P_0 represents the initial atmospheric pressure, at sea level.

a. At what elevation, to the nearest tenth of a meter, will the atmospheric pressure be a quarter of any initial amount?

b. Throwback: Can you rewrite the equation using a positive exponent?! ☺
(I believe you can!)

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3. Frank invested \$1000 in an account that earns 1.8% interest, compounded continuously according to the formula $A = P * e^{rt}$, where A is the amount of money in the account, P is the principal amount, r is the interest rate, and t is the number of years.

a. Approximately how many years will it take for Frank to double his money, assuming he does not add or withdraw from the account?

b. Frank has another bank account in which he deposited \$1000. This account also earns interest compounded continuously, but at a rate of 2.2%. If Frank made both of his initial deposits on January 1st, 2016, what will be the first year in which the amount of money in the first account exceeds the amount of money in this account?

★ **Example 3:** The population of town A was 2000 in the year 2010, and due to development of the land, has continued to grow exponentially according to the function $P(t) = 2e^{0.038t}$, which represents the population (in thousands) of the town t years after 2010.

a. Based on the model, what is the average rate of change in the population of the country from 2010 to 2015?

b. The equation $18 = 2e^{0.038t}$ could be used to determine when the population will reach 18,000. Write the solution of the equation as a single logarithm, and then determine how many years it will take for the population to reach 18,000.

c. Based on this model, in which years would the population of the country be more than 20,000? Circle all that apply.

2060	2070
2071	2072
2080	2061

d. The population of another town (town B) can be modeled by the function $N(t) = 1.5e^{-0.024t}$, where t is the number of years since 2010. Determine the first year in which the population of town B will exceed that of town A.

e. The population of another town (town C) can be modeled by the function $M(t) = 1.3e^{.045t}$, where t is the number of years since 2010. Determine the first year in which the population of town C will exceed that of town A.

✏ **Practice:** Solve each problem! Support your answers. Be sure to check for reasonableness also.

4. A scientist is observing bacteria in dish A, which are dying off at a rate of 25% each half an hour. There were initially 10 million microbes in the dish.

a. Write a function to model the number of microbes, N, in millions in dish A after t minutes.

b. Determine when the number of microbes in the dish will be half of the original amount, to the nearest minute.

c. Assuming the scientist began the observation at 5:00 AM, circle all times at which the number of microbes will be less than 3 million.

6:05 AM	6:06 AM
7:05 AM	7:06 AM

d. The scientist also observed a smaller dish of a different bacteria, beginning at 5:00 AM. The model she designed for the population of microbes, in millions, in this second dish (dish B) is $P(t) = 12 * 2^{-t/30}$. What will be the first minute in which dish B contains less microbes than dish A?

e. The scientist also observed a third dish of a different bacteria, beginning at 5:00 AM. The model she designed for the population of microbes, in millions, in this third dish (dish C) is $P(t) = 12 * (\frac{1}{2})^{-t/30}$. What will be the first minute in which dish B contains less microbes than dish A?

✏ **Mixed Practice:** Solve, check, and support your answers for each problem!

5. Some prehistoric cave paintings were discovered! The paintings contained 30% of the original carbon-14. Carbon-14 decays according to the equation $A = A_0 e^{-0.000121t}$, where time is measured in years. Approximately how old, to the nearest year, are the paintings?

6. Paula purchased a house in 2013 for \$500,000. The value of the house, V, fortunately grows exponentially according to the model $V(t) = 500000e^{.0453t}$, according to time in years.

a. Based on the model, determine when Paula can expect the house will to be worth over \$750,000.

b. Another house's value (also purchased in 2013) can be approximated using the model $V(t) = 200500e^{.0562t}$. Determine how much this house was purchased for, as well as the first year in which the house will be worth more than Paula's house. You may use desmos.com, and write your equations in terms of hundred-thousands.

c. Paula decided against purchasing a million-dollar home earlier in the year, since the house was appraised to decrease exponentially in value over time. Write any function, V(t) that could possibly model the value of this house. (There are infinite answers, since we do not know the exact rate of decrease!)

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7. Unfortunately due to construction of buildings, the number of deer in a certain area is decreasing exponentially at a rate of 15% per year. The deer population in this area was originally 700. Assuming the pattern continues, write an equation to model this situation, and determine how many years it will take for the deer population to become lower than 100 deer.

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8. Some medical tests involve a radioactive material, Technetium-99m, which decays exponentially according to the equation $A = A_0e^{-0.1035t}$, if time is measured in hours.

a. Determine the half-life of Technetium-99m.

b. If a procedure involves an injection that include Technetium-99m, will the radioactive material ever be completely diminished from the body? Explain. (You may want to show a graph and/or table of values to further support your response!)

Write down any important idea related to the problems you worked with today.

AND/OR

Write down any questions you still have regarding the problems to worked with today.

Homework:

p. 457 #50, 51, and 52

and the following three problems:

1. Some prehistoric cave paintings were discovered! The paintings contained only 14.3% of the original carbon-14. Carbon-14 decays according to the equation $A = A_0e^{-0.000121t}$, where time is measured in years. Approximately how old, to the nearest year, are the paintings?

2. Valeriya has three accounts, each of which involve interest compounded continuously according to the formula $A = P*e^{rt}$, where A is the amount of money in the account, P is the principal amount, r is the rate, and t is the number of years. The equations for the accounts are as follows:

Account A: $A = Pe^{0.025t}$
Account B: $A = Pe^{0.028t}$
Account C: $A = Pe^{-0.026t}$

a. One of these accounts represents money that Valeriya has already saved, and is currently using to pay off a loan. Which account is this, and how do you know?

b. State the interest rate for the other two accounts.

c. Valeriya opened all three accounts on the same day in 2015, and does not plan on depositing into or withdrawing money from the account for at least 40 years. In account A, she initially deposited $1500. On this day, Valeriya also deposited $800 into account B and $1300 into account C. Determine the first year when account C will have less money than account B.

d. Determine the first year in which account A will have less money than account C.

3. A certain bacteria triples in size every 20 minutes. An initial sample of the bacteria contains 10,000 bacteria.

a. Write an exponential equation to represent N, the total number of microbes (in thousands) as a function of x, the number of minutes the sample has been observed.

b. Approximately how many minutes will it take for there to be 500,000 microbes?