Proving Inverse Relations via Composition Homework

Directions: Be sure to show all work, communicate your thought process, and justify your reasoning. Remember to check that your answers are complete, correct, and reasonable.

Prove whether or not each pair of functions are inverse, via function composition.

1. $p(x) = \sqrt{x+5} - 2$
   $q(x) = (x+2)^2 - 5$

2. $r(x) = x^3 - 8$
   $s(x) = \sqrt[3]{x+8}$

3. $f(x) = \dfrac{x+4}{8}$
   $g(x) = 4x + 8$

4. $v(x) = 4x^3 + 7$
   $w(x) = \sqrt[3]{4x-7}$

5. $m(x) = \dfrac{x+6}{4}$
   $b(x) = 4x - 6$

6. $g(x) = (x-9)^2 + 3$
   $h(x) = \sqrt{x+9} - 3$

Solutions on next 2 pages…

1) $(p \circ q)(x) = \sqrt{(x+2)^2 - 5 + 5} - 2$

$= \sqrt{(x+2)^2} - 2 = x + 2 - 2$

$= x$ ✓

$(q \circ p)(x) = (\sqrt{x+5} - 2 + 2)^2 - 5$

$= (\sqrt{x+5})^2 - 5$

$= x + 5 - 5 = x$ ✓

$p(x)$ & $q(x)$ ARE inverses :)

2) $r \circ s(x) = (\sqrt[3]{x+8})^3 - 8 = x + 8 - 8 = x$ ✓

$s \circ r(x) = \sqrt[3]{x^3 - 8 + 8} = \sqrt[3]{x^3} = x$ ✓

$r \circ s(x) = s \circ r(x) = x$, so $s(x)$ & $r(x)$ are inverses!

3) $f \circ g(x) = \frac{4x+8+4}{8} = \frac{4x+12}{8} = \frac{x+3}{2} \neq x$

$f \circ g(x) \neq x$, so $f(x)$ & $g(x)$ cannot be inverses

(could also show $g \circ f(x) \neq x$:))

4) $r \circ w(x) = 4(\sqrt[3]{4x-7})^3 + 7$

$= 4(4x-7) + 7 = 16x - 28 + 7 = 16x - 21 \neq x$

$r \circ w(x) \neq x$, so $r(x)$ & $w(x)$ cannot be inverses

(could also show $w \circ r(x) \neq x$:))

5) $m \circ b(x) = \frac{4x-6+6}{4} = \frac{4x}{4} = x$ ✓

$b \circ m(x) = 4\left(\frac{x+6}{4}\right) - 6 = x + 6 - 6 = x$ ✓

$m \circ b(x) = b \circ m(x) = x$, so $m(x)$ & $b(x)$ are invers

6) $(g \circ h)(x) = (\sqrt{x+9} - 3 - 9)^2 + 3$

$= (\sqrt{x+9} - 12)^2 + 3 \neq x$

also

$(h \circ g)(x) = \sqrt{(x-9)^2 + 3 + 9} - 3$

$= \sqrt{(x-9)^2 + 12} - 3 \neq x$

so $g(x)$ and $h(x)$ are NOT inverses!