Algebra I Skills Review Sessions

Please work slowly to review these concepts/skills before being assessed on them. Please go through the Summer Packet one section at a time, reaching each of the goals below. An additional copy of the packet and solutions can be found on our class website under the "Stay Up to Date" section.

Complete all work neatly on scrap paper, as staying organized will set you up for success.
Please ask me questions, and I will guide you to an understanding and/or show you an example! ☺

★ **Goal #1:**
If you have not done so already, please check your answers using the provided keys, and write down any questions you have. Please make sure to ask me your questions and/or discuss the questions you have with your group! ☺ You may also "post" a question on the board using your post-it notes.

I strongly suggest you **retry** any problems you did not answer correctly, being sure to explain what your error was and how/why you fixed it as such according to the mathematical concepts required for solving. Also please keep in mind that in mathematics, oftentimes there are more than one correct solution strategies.

★ **Goal #2**
~ Try some of the **additional practice** problems within each section. Be sure to take note of any important ideas, check your answers, and ask your group members and myself questions .

OR

~ Create some problems that could be solved using the concepts/ideas within each section. Be sure to share these with me.

Practice Problems: p. 1 – 5 **Teacher's Tips: p. 6-9** **Solutions: p. 10-12**

A and B. Number Classifications and Order of Operations

Extra Practice Problems!

1. Classify each real number as an integer, whole number, natural number, rational number, and/or irrational number. State all that apply.

a. -10 b. -2.3 c. 0 d. 18 e. $\sqrt{5}$ f. $\sqrt{9}$

2. Decide whether each statement is sometimes, always, or never true.

a. A natural number is rational.
b. An integer is natural.
c. A number is both natural and whole.
d. An irrational number is an integer.
e. A never-ending decimal number is rational.

3. Evaluate each expression.

a. $3 + 8|2 - 6|$ b. $(-6)^3 \div 3 * 2$ c. $2|-8 + 5|^2 + 3q^2$ if $q = -9$

C. Simplifying Algebraic Expressions

Extra Practice Problems!

1. Simplify each expression.

a. $8p - 13(2p - m)$

b. $(8r + 7)(4r - 12)$

c. $13s – 28t + 4t(3 - s) – st$

d. $8x^2 – 12x^2$

e. $9v^3(v^2 + vg – 2g + 8) – 3vg(4v^3g - 4vg + 1)$

2. Write an algebraic expression for each situation.
a. two less than the quotient of a number and five
b. the product of seven and a number, totaled with nine
c. twenty more than the product of a number and three
d. the difference of a number and triple the number

D. Solving Basic Equations

Extra Practice Problems!

Solve each equation.

1. $8m – 7 = 41$

2. $9x + 20 = -16x – 5$

3. $-4(x + 9) = -8x – 4$

4. $-2+ 5(2y – 8) = 8(-2 + 3y) – 4y$

5. $9(2x - 8) = -5(3x + 5)$

6. $\frac{2x+1}{-2} = \frac{12x-3}{4}$

E. Solving & Graphing Solutions to Basic Inequalities

Extra Practice Problems!

Solve each inequality, and graph the solution on a number line.

1. $2(q – 1) + 10 < 30$

2. $-7k > - 343$ and $-5k + 5 < 80$

3. $4 + 5x < x + 3 < 9x + 10$

4. $-18 > x – 3 < 20x$

5. $6(3x – 1) < 36$ or $-8(2x + 1) < 64x$

6. $13 < -8p – 10p < 54$

7. $-7(5g – 2) + 2 > -5g – 7(g + 1)$

8. $6 + w < -2(w + 5) <22$

9. Rewrite the inequality so that the variable y is isolated: $\frac{1}{2}x - 4y > -128$.

F. Solving Basic Word Problems

Extra Practice Problems!

1. In an animal shelter, there are 10 more dogs than cats and half as many birds as there are cats. The animal shelter has a total of 25 animals (only dogs, cats and birds). Determine how many of each animal are at the shelter.

2. A rectangular pool is going to built in such a way that the width is twelve feet less than double the length, and the length is no less than five feet. The pool will must have a perimeter less than 96 feet. Determine the range of possible lengths for the pool. Express your answer as a compound inequality, and explain the inequality via a short sentence.

4. You are planning on making a donation to your favorite charity. Your friend tells you that he/she will add an additional 40% to your donation! How much should you donate in order that the charity will receive at least $100?

5. Adam is two years younger than Joanna. Joanna is three years older than double Katie's age. The sum of Adam, Joanna, and Katie's ages is 59. How old is Adam?

G. Fraction Operations

Extra Practice Problems!

1. Simplify each expression.

a. $\frac{5}{8} + \frac{2}{5} - \frac{1}{2}$

b. $2\frac{2}{3} - \frac{1}{4}$

2. Solve each problem.

a. Your goal is to complete 5½ miles of cardio at the gym today. If you ran for 1¾ of a mile, cycled for two-thirds of a mile, and used the elliptical for seven-tenths of a mile, how many more miles of cardio should you complete to reach your goal?

b. You also used the step machine for what would be equivalent to another three-fifths of a mile! How many miles do you have left to complete?

3. Simplify each expression.

a. $\frac{3}{8} * \frac{7}{9} * \frac{1}{2}$

b. $8\frac{1}{3} * \frac{2}{3}$

4. Solve each problem.

a. Joe runs four-fifths of a mile every nine minutes! How far can Joe run in an hour minutes, assuming he maintains a steady pace?

b. You are baking rice crispie treats for your friend's birthday. The recipe calls for 3¼ cups of marshmallows. If you need two and a half batches, how many cups of marshmallows will you need?

c. Considering the situation from part b, each bag of marshmallows contains 4 cups. If each bag costs $1.49 and there is no tax, how much money should you expect to spend on marshmallows?

5. Simplify each expression.

a. $\dfrac{1/8 \div 4}{3/2}$

b. $\dfrac{7}{3} \div 2\dfrac{1}{4}$

6. A recipe for 32 pancakes calls for 3½ cups of flour. You only want to make 24 cookies. How much flour should you use?

H. <u>Linear Functions</u>

1. What is the slope between the points (3, 8) and (-2, 12)? Show and explain all work.

2. What is the slope between the points (-7, 2) and (-3, 5)? Show and explain all work.

3. Determine the slope of each line.

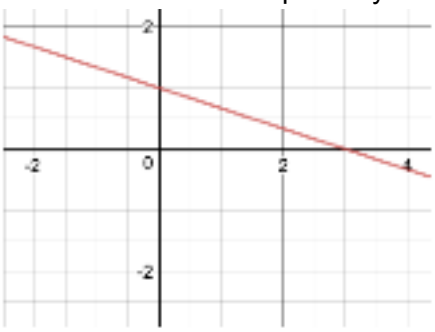
a. y = x – 9

b. -3y = 2x + 1

c. y = -8

d. 5x = y - 2

4. Determine the x-intercept and y-intercept of each line.

a. 

b. 3y + 4x = 24

5. A line is represented as the equation y = -¼x – 2. What is the slope of a parallel line?

6. A line is represented as the equation y = -3x + 5. What is the slope of a perpendicular line?

7. Graph each line on graph paper.

a. 3y = -2x + 9

b. x = 3

I. Factoring Polynomials

1. Factor out the GCF (greatest common factor).

a. $7y^3 - 28y$ b. $10m^3n^4 + 5mn - 30m^2n$ c. $8x^6y^8 - 12xy^2 + 20x^3yz$

2. Factor each binomial.

a. $x^2 - 16$ b. $x^2 - 144$ c. $81x^2 - 36$

3. Factor each trinomial. (Look for a GCF first ☺)

a. $3x^2 - 300$ b. $b^2 - 5b - 24$ c. $2r^2 - 26r + 60$

J. Exponent Properties

Simplify each expression using the properties of exponents.

a. $8wz \cdot 5w^2z^8$ b. $(9p^3q)^2$ c. $\dfrac{6v^3z^3}{5z^3}$

d. $(m^3)^4$ e. $(5d^2f^0)^3$ f. $\dfrac{3h^3g^7}{12gh^8}$

g. $\dfrac{a^3b^2c}{a^2b}$ h. $10p^2q^3 \cdot 3p \cdot 2q^7$ i. $\left(\dfrac{18m^3n^{20}p^3 \cdot 2mnpq}{40p^{18}q^3n \cdot 2m}\right)^0$

Teacher's Tips:

*Dividing by zero is undefined (if you have $20 to divide among 0 people, what happens to the money?!), but zero divided by any number is just 0 (if you have $0 to divide amongst 20 people, each person receives $0).

Real Number System:

- A number is rational if you can express it as a ratio or fraction.
- A synonym for irrational is "crazy;" if it would be crazy for you to sit and list all of the digits of a number (that is, it never ends or never repeats), then the number is irrational.
- Natural numbers are how you naturally count.
- Whole numbers are the natural numbers and "the hole," 0.
- Integers are all of the whole numbers and their opposites.

Order of Operations:

- Use GEMDAS.

G: Grouping; E: Exponents; MD: Multiplication/Division (in order of appearance from left to right); AD: Addition/Subtraction (in order of appearance from left to right)

- Evaluate what is in the absolute value bars or parentheses or any grouping symbols first.
- Also please be aware that an expression such as -2|-9| becomes -2(7), which means "-2 times 9."
- If the expression looked something like |9 – 10|, this would mean to take the absolute value of 9 minus 10. This would become the absolute value of -1, which is 1.
- An expression like 4 - |-3| means "four minus the absolute value of -3," which would be 4 – 3, or -1.
- Remember that absolute value is a number's distance from zero on a number line, and therefore can never be negative.
- Do not distribute into absolute value bars. For example, -2|-7 – 10| does NOT mean -2*-7 + -2*-10. Simplify inside the absolute value bars first, then distribute, to end up with -2|-17|, or -2 * 17, so -34.
- The reciprocal is the same as the multiplicative inverse.
- For example, to determine the reciprocal of -2½, first convert to a mixed number. -2½ is -5/2. The multiplicative inverse (reciprocal) of this would be -2/5 because (-5/2)*(-2/5) = 1.

Evaluating Expressions:

- Substitute in the given values first. Then evaluate. It is often helpful and more efficient to simplify what is in parentheses before applying the multiplication.
- Example:, if you are asked to evaluate $-3x^2$ for x = -2, you would have $-3(-2)^2$, which becomes -3(4), which means to multiply -3 by positive 4, resulting in -12.
- Example: $-8x^2$ reads "-8 times x squared, or the product of -8 and the number squared."

So if x were 2, $-8x^2$ would be -8 * 4, or - 32

Simplifying Expressions:

- Example: 5^2 means 5*5, $(8x)^2$ means (8x) * (8x), $(x – 3)^2$ means (x – 3)(x – 3).
- Exponents must be applied before multiplication is applied.
- Be careful to keep track of and double check your signs.
- Consider the expression $(3 + 4)^2$. Applying the order of operations, this becomes 7^2, which is 49. Another correct way to write this expression would be (3 + 4)(3 + 4), which is 7*7, or 49.

If we were to distribute the squared into the parentheses, we would end up with a result of $3^2 + 4^2$, which is 9 + 16, or 25. This is missing out on the middle terms, since the actual value should be 49.

Word Problems:

- Consecutive means in a row. For example, 4, 5, and 6 are consecutive integers.
- Let the unknown first number/value be "x." Write an expression in terms of x for each of the other two numbers. Then write an equation combining the three numbers so that they sum to the requested value.
- Identify what is unknown first. Let the unknown value that you know the least about be your variable. Use this variable to write an expression for the other unknown variable. Then write and solve an equation.
- For example, if I know an animal shelter has 10 more cats than dogs and 26 total animals, I would let d be the number of dogs at the shelter (since this is the unknown we know the least about). Then I could go on to say that the number of cats must be d + 10. We know that cats + dogs = 26, so d + 10 + d = 26. This results in 2d = 16, which gives us d = 8. Therefore there are 8 dogs, and 8 + 10, or 18, cats at the animal shelter. Check that this satisfies the conditions of the problem: 8 dogs + 18 cats = 26 total animals.

Inequalities:

- Apply the distributive property first. Be sure to check each step, keep track of your signs, and organize your work neatly.
- When dividing or multiplying by a negative number, remember to flip the direction of the inequality symbol.
- Break inequalities like $-3 < x + 1 < 9$ into two smaller inequalities: $-3 < x + 1$ AND $x + 1 < 9$.
Solve and graph BOTH of these.
- Solve and graph BOTH of the separate inequalities when you see the word "or."

Fractions:

- When adding/subtracting fractions, a common denominator is required. Then, the problem becomes simple because for example, one-fourth plus two-fourths equals three-fourths.
- When multiplying fractions, multiply straight across. Multiply the numerators together to produce the new numerator, and multiply the denominators together to produce the new denominator.
- Be sure to simplify as much as possible.

Factoring:

- Always try to factor out a GCF first.
- Know your perfect squares from 1^2 to 20^2 (this will be EXTREMELY helpful throughout the rest of the year as well, and will make your experience in this class MUCH more smooth and enjoyable)
- Check your factoring using "FOIL."

Exponents:

- If you forget a rule, write out the expression in expanded form and see what happens.
For example, $(x^3)^2$ means $x^3 * x^3$ or $x * x * x * x * x * x$, which is x^6
- ANY expression raised to the zero power is 1.

Linear Functions:

~ Determining Slope

$$Slope = m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

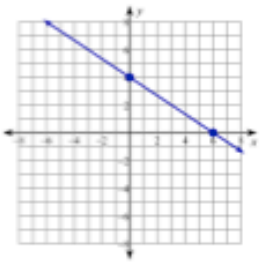
Line with **positive** slope: ↗

Line with **negative** slope: ↖

Line with **zero** slope: ↔ (0 rise)

Line with **undefined** slope: ↕ (0 run)

Example using a graph:



1) Start at a lattice point.
2) Determine "rise" and "run" to reach another lattice point.

m $= -\frac{4}{6} = -\frac{2}{3}$.

Example given two points:

$(12,-14),(12,17)$ **Use formula:**

$\frac{17-(-14)}{12-12} = \frac{31}{0}$ **undefined (vertical line)**

~ Slope-Intercept Form

$y = mx + b$

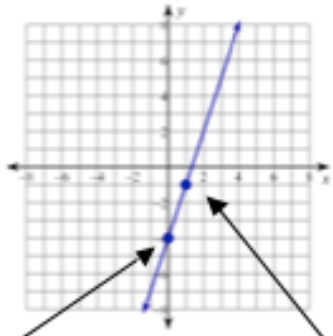
(slope) (y-coordinate of y-intercept)

Example: The line whose equation is $y = 5x - 2$ has a slope of 5 (meaning $\frac{5}{1}$), and a y-intercept of $(0,-2)$.

~ Graphing lines from slope-intercept form

To graph a line, you must have (or be able to find) two points on the line. The y-intercept is one point, and that is where we can start. To find a second point, use the slope of the line (rise over run). Connect the two points, extending through the whole graph, and use arrows on both ends (since the line is infinite).

Example: Let's graph $y = 3x - 4$. The y-intercept is $(0,-4)$ so you must go **down** to -4 on the **y**-axis:



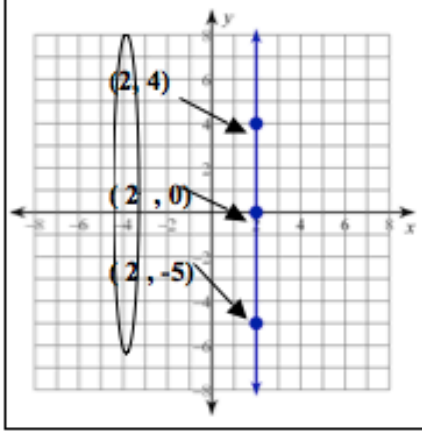
Start here

Use slope to get here

~ Horizontal and Vertical Lines

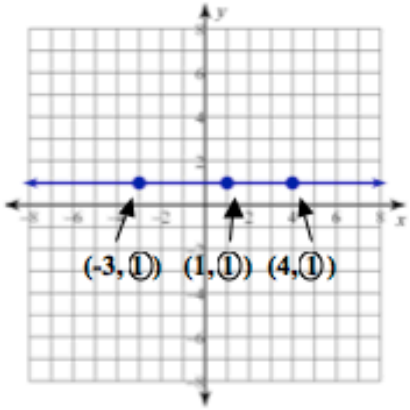
Vertical

Lines such as x = 2 are vertical lines. You can think of this equation as the line where all of the ***x-values*** *of the points are 2.*



Horizontal

Lines such as y = 1are horizontal lines. you can think of this equation as the line where all of the ***y-values*** *of the points are 1.*

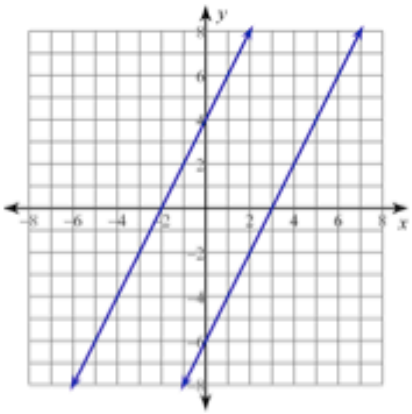


~ Parallel and Perpendicular Lines

Parallel

Parallel lines have the same slope.

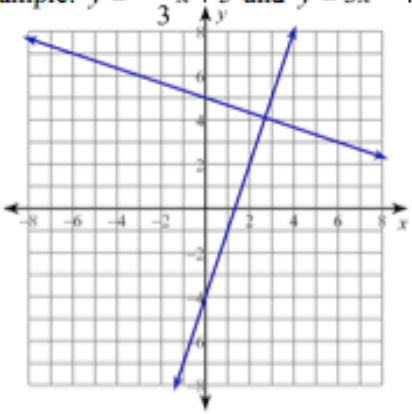
Example: $y = 2x - 6$ and $y = 2x + 4$



Perpendicular

Perpendicular lines have slopes that are opposite reciprocals.

Example: $y = -\frac{1}{3}x + 5$ and $y = 3x - 4$



- Slope is the change in y-values divided by the change in x-values, or rise over run
- The x-intercept is when the graph crosses the x-axis, when the y-value is zero (no "rise").
- The y-intercept is when the graph crosses the y-axis, when the x-value is zero (no "run").
- y = mx + b is slope intercept form, where m is the slope and the point (0, b) is the y-intercept.
- Isolate y first to see the slope-intercept form of a line
- First plot the y-intercept, then from there use the rise/run to plot at least 2 more points. You can also graph BOTH intercepts and connect them, if this is transparent in the equation, such as 2x + 4y = 8. The x-intercept would be (4,0) and the y-intercept would be (0, 2).
- Parallel lines have the same slope (ex. 4 and 4). Perpendicular lines have opposite reciprocal slope (ex -1/3 and 3).
- Equations in the form y = a number represent horizontal lines, which have a slope of 0.
- Equations in the form x = a number represent vertical lines, which have undefined slope.