Name: ______________________________ Date: ______________________ **Unit 7 Class Work**

Solving Exponential Equations (Applied Problems) Class Work

Objective: *You will be able to solve problems involving exponential situations.*

★ **Quick Review:** Exponential Growth & Decay Equations

$A = A_0(1+r)^t$ Growth

Decay $A = A_0(1-r)^t$

A: amount

A_0: initial amount

r: rate (as a decimal)

t: time

★ **Example 1:** A radioactive substance decays at a rate of 3.5% per hour. Consider a sample of 250 mg of a radioactive substance. initial

a. Write an exponential equation to represent A, the amount of the substance, as a function of t, the number of [redacted] hours the sample has been observed.

b. How many [redacted] hours, to the nearest tenth of [redacted] an hour will it take for the substance to decay to less than 10 mg?

$A = ?$

$A_0 = 250$

$r = \frac{.035}{hr}$

$A = 250(1-.035)^t$

$\boxed{A = 250(.965)^t}$

$\frac{10}{250} = \frac{250(.965)^t}{250}$

$\frac{1}{25} = .965^t$

$\frac{\log(\frac{1}{25})}{\log .965} = \frac{t\cancel{\log(.965)}}{\cancel{\log(.965)}}$

$t \approx 90.3488$

90.3 hours

Practice: Solve each problem! Support your answers. Be sure to check for reasonableness also.

1. A scientist has a sample of bacteria that initially contains 20 million microbes. She determines that the number of bacteria increases by 50% every hour!

a. Write an exponential equation to represent N, the total number of microbes (in millions) as a function of x, the number of [redacted] hours the sample has been observed.

b. Approximately how many [redacted] hours will it take for there to be 200 million microbes?

$N = 20(1+.50)^x$

$N = 20(1.5)^x$

$\frac{200}{20} = \frac{20(1.5)^x}{20}$

$10 = 1.5^x$

$\frac{\log 10}{\log 1.5} = \frac{x\log(1.5)}{\log 1.5}$

≈ 5.7 hours

★ **Example 2:**

The formula $P = P_0 * 2^{-h/4795}$ can be used to determine the atmospheric pressure, P, in millimeters of mercury, at an elevation of h meters above sea level. P_0 represents the initial atmospheric pressure, at sea level.

At what elevation, to the nearest tenth of a meter, will the atmospheric pressure ~~be a quarter of any initial amount~~?

use .25 for final value

; 1 for initial

$P = P_0 \cdot 2^{-h/4795}$

$.25 = 2^{-h/4795}$

$\left(\frac{\log(.25)}{\log(2)}\right) = \frac{\cancel{x}h}{4795}\ \cancel{\log(2)}$

$\cdot 4795 \qquad \cdot -4795$

$\boxed{h \approx 9{,}590 \text{ meters}}$

Practice: Solve each problem! Support your answers. Be sure to check for reasonableness also.

The isotope Plutonium-240 can be used to determine the age of fossil samples. Plutonium-240 has a half-life on 6,563 years (meaning its mass decreases by half every 6,563 year).

a. Write a function M(t) to represent the mass of Plutonium-240 in a sample that is t years old, assuming the original mass is M_0 grams. Represent the function once using a positive exponent, and once using a negative exponent.

$$M(t)=M_0\left(\frac{1}{2}\right)^{t/6563}$$

$$M(t)=M_0(2)^{-t/6563}$$

b. Is M(t) linear or nonlinear?

Non linear

c. Does the rate of change of M(t) increase or decrease over time?

decreasing

d. A sample is found to have one-eighth of the original amount of Plutonium-240. How old is the sample?!

$M_0 = 1$

Final $M = 1/8$

$$\frac{1}{8} = 2^{-t/6563}$$

$$\frac{\log(1/8)}{\log(2)} = -t/6563 \cdot \frac{\log(2)}{\log(2)}$$

$$\cdot 6563 \qquad \cdot 6563$$

$t \approx 19,689$ years

✐ **Practice:** Solve each problem! Support your answers. Be sure to check for reasonableness also.

4. A certain element decays fossils, and helps us determine the fossil's age. The half-life of the element is 3,000 years.

a. Write a function, M(x) for the mass of the element at any age x.

$$M(x)=M_0\left(\frac{1}{2}\right)^{x/3000}$$

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b. Is M(x) linear or non-linear, and does the rate of change increase or decrease over time?

Non-linear
decreasing

$$M(x)=M_0\left(\frac{1}{2}\right)^{t/3000}$$

b. If a fossil is found to have one-fourth of the initial amount of the element, how old is the fossil?

$$\frac{1}{4}=\left(\frac{1}{2}\right)^{t/3000}$$

$$\frac{\log\left(\frac{1}{4}\right)}{\log\left(\frac{1}{2}\right)} = \frac{t}{3000}$$

$$\cdot 3000 \qquad \cdot 3000$$

$t \approx 6000$ yrs old

✐ **Mixed Practice:** Solve, check, and support your answers for each problem!

5. Some prehistoric cave paintings were discovered! The paintings contained 30% of the original carbon-14. Carbon-14 decays according to the equation $A = A_0e^{-0.000121t}$, where time is measured in years. Approximately how old, to the nearest year, are the paintings?

6. Paula purchased a house in 2013 for \$500,000. The value of the house, V, fortunately grows exponentially according to the model $V(t) = 500000e^{.0453t}$, according to time in years.

Homework:

p. 457 #50, 51, and 52

and the following five problems:

1. Some prehistoric cave paintings were discovered! The paintings contained only 14.3% of the original carbon-14. Carbon-14 decays according to the equation $A = A_0e^{-0.000121t}$, where time is measured in years. Approximately how old, to the nearest year, are the paintings?

2. Valeriya has three accounts, each of which involve interest compounded continuously according to the formula $A = P*e^{rt}$, where A is the amount of money in the account, P is the principal amount, r is the rate, and t is the number of years. The equations for the accounts are as follows:

Account A: $A = Pe^{0.0025t}$
Account B: $A = Pe^{0.0028t}$
Account C: $A = Pe^{-0.026t}$

a. One of these accounts represents money that Valeriya has already saved, and is currently using to pay off a loan. Which account is this, and how do you know?

b. State the interest rate for the other two accounts. $1.0 - .038 = .962$

3. A new Acura is valued at $39,000, but its value depreciates 3.8% per year. Write a function, v(t) to represent the value of the car after t years. $v(t) = 39000(.962)^t$

You would like to sell the car before it is worth less than $10,000? What is the maximum amount of years you should wait before selling the car? $\frac{10}{39} = \frac{39}{39}(.962)^t$

4. An antique item was purchased in 2011 for $450, and was appraised as having a value that would increase by 8% each year.

a. Write a function V(t) to represent that value, V, of the item after t years since its appraisal.

b. By how much money did the value of the item increase in the first year?

$$\frac{\log\left(\frac{10}{39}\right)}{\log .962} = t\,\frac{\cancel{\log .962}}{\cancel{\log .962}}$$

1. Some prehistoric cave paintings were discovered! The paintings contained only 14.3% of the original carbon-14. Carbon-14 decays according to the equation $A = A_0e^{-0.000121t}$, where time is measured in years. Approximately how old, to the nearest year, are the paintings?

$A = .143$
$A_0 = 1$

$.143 = e^{-.000121t}$

$$\frac{\ln .143}{-.000121} = \frac{-.000121t \cancel{\ln e}}{\cancel{-.000121}}$$

$t \approx 16,074$ years old

May 9-8:17 AM

c. What is the value of the item currently?

d. Does the rate of change of the value of the item increase, decrease, or maintain over time?

e. How long will it take the item to be worth at least $2000?

5. Potassium-40 is a common isotope found in nature. Its half-life is 1,280,000,000 years. Write a function M(t) to determine the mass of Potassium-40 found in any rock/mineral. Then determine how old a rock/mineral that has one-fourth of the amount of its initial Potassium-40 is.

Write down any important idea related to the problems you worked with today.

AND/OR

Write down any questions you still have regarding the problems to worked with today.