

### Even and Odd Polynomial Functions Class Work

✂ **Objective:** You will be able to identify & prove even and odd polynomial functions.

Consider a given function  $f(x)$ , as well as the reflections  $f(-x)$  and  $-f(x)$ .

If one pair of the functions are equal, then  $f(x)$  is even.

Hypothesize: Which pair do you think would have to be equal for the function to be even? Why? Hint: Consider the graph of the functions.

★ **Even functions:**

- A function is considered even when

\*In other words, the graph of an even function is

- Prove that  $x^2 - 4$  is even.

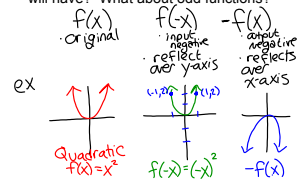
What comes to mind when you think of the word "even?" What about "odd?"

- divisible by 2
- the same on both sides
- not equally divide by 2
- not exactly equal

What qualities do you think something that is even has? What about something that is odd?

- equal/the same

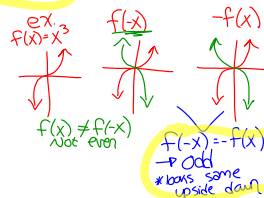
What qualities do you think even functions will have? What about odd functions?



★ **Even:**

- same on left/right
- reflection over y-axis does not change the graph
- $f(x) = f(-x)$

ex:  $f(1) = 2$   $f(-1) = 2$   
 $f(x) = x^2$   
 $f(-x) = (-x)^2 = x^2 = f(x)$



What comes to mind when you think of the word "even?" What about "odd?"

Even  
divisible by 2  
"same on both sides"  
odd  
"weird"  
not divisible by 2

What qualities do you think (something) that is even has? What about something that is odd?

What qualities do you think even functions will have? What about odd functions?

$f(x)$  original  
 $f(-x)$  reflection over y-axis  
 $-f(x)$  reflection over x-axis

Quadratic  $f(x) = x^2$   
 $f(x) = x^2$   
 $f(-x) = (-x)^2 = x^2$   
 $f(x) = f(-x)$   
(Even)  
symmetric about y-axis

ex.  $f(x) = x^3$   
cubic  
 $f(-x) = (-x)^3 = -x^3$   
 $-f(x) = -x^3$   
reflection over y-axis is the same as reflection over x-axis

Nov 2-8:13 AM

What comes to mind when you think of the word "even?" What about "odd?"

What qualities do you think something that is even has? What about something that is odd?

What qualities do you think even functions will have? What about odd functions?

Nov 2-8:13 AM

Consider a given function  $f(x)$ , as well as the reflections  $f(-x)$  and  $-f(x)$ .

If one pair of the functions are equal, then  $f(x)$  is even.

Hypothesize: Which pair do you think would have to be equal for the function to be even? Why? Hint: Consider the graph of the functions.

★ Odd functions:

- A function is considered odd when

\*In other words, the graph of an odd function is

- Prove that  $x^3 - x$  is odd.

★ Note that some functions are neither odd nor even!

Practice!

In your notebook...

## EVEN

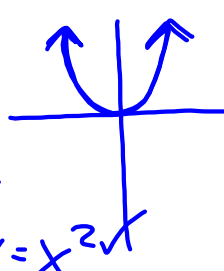
Visually

symmetric  
about y-axis

Algebraically

$$f(x) = f(-x)$$

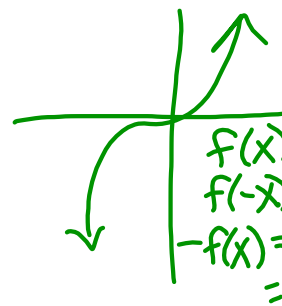
Example

$$\begin{aligned} f(x) &= x^2 \\ f(-x) &= (-x)^2 \\ &= -x \cdot -x = x^2 \checkmark \end{aligned}$$


## ODD

symmetric  
about origin

refl. over is refl.  
x-axis same y-axis  
as  
 $-f(x) = f(-x)$


$$\begin{aligned} f(x) &= x^3 \\ f(-x) &= -x^3 \\ -f(x) &= -(x^3) \\ &= -x^3 \checkmark \end{aligned}$$

Name: \_\_\_\_\_ Date: \_\_\_\_\_

1. Identify each function as even, odd, or neither. Justify your answers completely.

a.  $r(x) = (x+1)^2 = (x+1)(x+1) = x^2 + 2x + 1$  not even

$r(-x) = (-x+1)(-x+1) = x^2 - 2x + 1$

$-r(x) = -(x^2 + 2x + 1) = -x^2 - 2x - 1$

$r(-x) \neq -r(x)$  not odd

$r(-x) \neq r(x)$  neither

b.  $t(x) = x^3 + 2x$

$t(-x) = (-x)^3 + 2(-x) = -x^3 - 2x$

$-t(x) = -(x^3 + 2x) = -x^3 - 2x$

$t(-x) = -t(x)$  odd

c.  $g(x) = x^5 - 3x^3 + 4x^2$

$g(-x) = (-x)^5 - 3(-x)^3 + 4(-x)^2 = -x^5 + 3x^3 + 4x^2$  not even

$-g(x) = -(x^5 - 3x^3 + 4x^2) = -x^5 + 3x^3 - 4x^2$

$g(-x) \neq -g(x)$  not odd

$g(-x) \neq g(x)$  neither

d.  $f(x) = 2x^4 - 3x^2$

$f(-x) = 2(-x)^4 - 3(-x)^2 = 2x^4 - 3x^2$

$f(-x) = f(x)$  even

e.  $n(x) = x^6 - 2x^2 - x$

$n(-x) = (-x)^6 - 2(-x)^2 - (-x) = x^6 - 2x^2 + x$

$n(-x) \neq n(x)$  not even

$-n(x) = -(x^6 - 2x^2 - x) = -x^6 + 2x^2 + x$

$n(-x) \neq -n(x)$  not odd

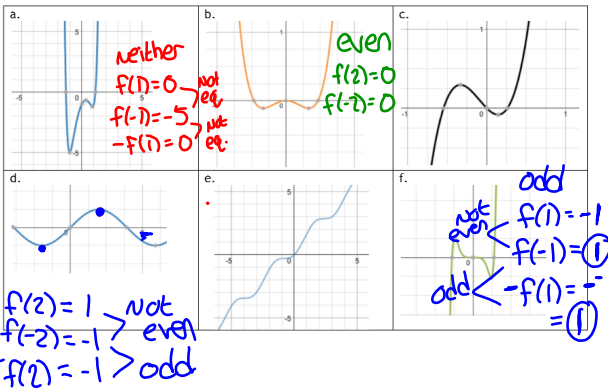
$n(-x) \neq n(x)$  neither

$$w(x) = x^7 - 3x^3 + x$$

Create any even function:  
prove that it  
is even.

**Practice Identifying Even vs. Odd Provided With the Graphs of Functions:**

Identify each function as even, odd, or neither. Justify your choice in detail, and be sure to mention an example or counter-example to support each each.



Complete the following OUT OF ORDER. ☺

- ★ Create a function equation or a graph for an even function.
- ★ Create a function equation or a graph for an odd function.
- ★ Create a function equation or a graph for a function that is neither even nor odd.

\*Switch with a partner, and determine the evenness or oddness of their functions!