Properties of Logarithms Class Work

❧ **Objective:** *You will be able to simplify and expand logarithms.*

★ **When the bases of logarithms are identical, the following properties hold...**

✴ $\log_b M + \log_b N =$ $\log_b (M \cdot N)$

Example: $\log_3 (8x) + \log_3 (2x) = \log_3 (16x^2)$

✴ $\log_b M - \log_b N =$ $\log_b \left(\dfrac{M}{N}\right)$

Example: $\log_2 (20x^5) - \log_2 (4x) = \log_2 (5x^4)$

✴ $\log_b M^a =$ $a \log_b (M)$

Example: $\log_4 (x^3) = 3\log_4 (x)$

$\text{b/c } \log_4 (x \cdot x \cdot x) = \log_4 x + \log_4 x + \log_4 x = 3\log_4 x$

★ **Can you create a rule for the following logarithms, for any given values x and y?**

✴ $\log_x x =$ ① $\qquad\qquad$ ✴ $\log_y 1 =$ ⓪

$x^? = x \qquad\qquad\qquad\qquad y^? = 1$

"log" once

✐ **Practice:** Write an equivalent expression to <u>condense</u> each logarithm.

1. $\log_3 2x + \log_3(x-4)$

$\log_3(2x \cdot (x-4))$

$\boxed{\log_3(2x^2 - 8x)}$

2. $\log_{12}17 - \log_{12}(2x+5)$

$\log_{12}\left(\dfrac{17}{2x+5}\right)$

exponents first

3. $\log_2 4^x$

$= \boxed{x\log_2(4)}$

4. $3\log(xyz^3) + \log(y)$

$\log(x^3y^3z^3 \cdot y)$

$\boxed{\log(x^3y^4z^3)}$

5. $\log(p) \overset{\bullet}{+} \log(q) \overset{\div}{-} \log(r)$

$\log\left(\dfrac{pq}{r}\right)$

6. $4\log(xy) - 6\log(y) + \log(x)$

$\log\left(\dfrac{x^4y^4 x}{y^6}\right)$

$= \boxed{\log\left(\dfrac{x^5}{y^2}\right)}$

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7. $\log_7 b - 5(2\log_7 c + \log_7(3d))$

$\log_7 b - (10\log_7 c + 5\log_7(3d))$

$= \log_7 \dfrac{b}{c^{10}(3d)^5}$

$= \boxed{\log_7\left(\dfrac{b}{243c^{10}d^5}\right)}$

8. $\ln(6x^2) - \ln(2x)$

$\ln\left(\dfrac{6x^2}{2x}\right)$

$\boxed{\ln(3x)}$

9. $\ln(4x)^2 + \ln(2) - \ln(x)$

$\ln\left(\dfrac{16x^2 \cdot 2}{x}\right)$

$= \boxed{\ln(32x)}$

10. $\log(3w) - 2(3\log(v) + \log(3t))$

$\log(3w) - (6\log v + 2\log(3t))$

$\log\left(\dfrac{3w}{v^6(3t)^2}\right)$

$\log\left(\dfrac{3w}{9v^6t^2}\right)$

$\log\left(\dfrac{w}{3v^6t^2}\right)$

11. $\log_9 \frac{1}{3} + 4\log_9 3$

$\log_9 \left(\frac{1}{3} \cdot 3^4 \right)$

$\log_9 \left(\frac{81}{3} \right) = \boxed{\log_9(27)}$

12. $\log_{16}(\frac{1}{4}) + 3\log_{16}4$

$\log_{16} \left(\frac{1}{4} \cdot 4^3 \right)$

$\log_{16}(16) = \boxed{1}$

$16^x = 16$

13. $4\log2 + \log7 - \log4$

$\log \left(\frac{2^4 \cdot 7}{4} \right) = \log \left(\frac{16 \cdot 7}{4} \right)$

$= \boxed{\log(28)}$

14. $\ln8 - 3*\ln2$

$\ln \left(\frac{8}{2^3} \right) = \ln \left(\frac{8}{8} \right)$

$= \boxed{\ln(1)}$

What if you had...

$\frac{1}{2}\log(81x) + \frac{1}{3}\log(8y^9)$

$\log(81x)^{1/2} + \log(8y^9)^{1/3}$

$\frac{\sqrt{81x}}{9\sqrt{x}}$

$\frac{\sqrt[3]{8y^9}}{2y^{9/3}}$

$2y^3$

$\log(9\sqrt{x}) + \log(2y^3)$

$\log(18y^3\sqrt{x})$

☆ #4: Make both $\log_4$ ☺

$$\boxed{77} \frac{2\log_b x}{3} + \frac{3\log_b y}{4} - 5\log_b z$$

$$\frac{2}{3}\log_b x + \frac{3}{4}\log_b y - 5\log_b z$$

$$\log_b\left(\frac{x^{2/3}\, y^{3/4}}{z^5}\right) = \log_b\left(\frac{\sqrt[3]{x^2}\cdot \sqrt[4]{y^3}}{z^5}\right)$$

Expanding

ex. 1

$$\log_3(2xy^2z^3)$$

$$\log_3(2) + \log_3 x + \log_3 y^2 + \log_3 z^3$$

$$\log_3(2) + \log_3 x + 2\log_3 y + 3\log_3(z)$$

ex. 2

$$\ln\left(\frac{8c}{d^3}\right) = \frac{\ln(8) + \ln(c) - \ln(d^3)}{\ln(8) + \ln c - 3\ln d}$$

ex 3.

$$\log(\sqrt{xyz}) = \log x^{1/2} + \log y^{1/2} + \log z^{1/2}$$

$$= \tfrac{1}{2}\log x + \tfrac{1}{2}\log y + \tfrac{1}{2}\log z$$

$$\tfrac{1}{2}(\log x + \log y + \log z)$$

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ex. 4

$$9\log_7\left(\frac{4 \cdot \sqrt[3]{y}}{x}\right)$$

$\downarrow$

$9\left(\log_7(4) + \log_7 y^{1/3} - \log_7 x\right)$

$9\left(\log_7(4) + \frac{1}{3}\log_7 y - \log_7 x\right)$

$9\log_7(4) + 3\log_7 y - 9\log_7 x$

$\text{⑤} \quad \log_2\left(5 \cdot (3y+1)^4\right)$

$\log_2 5 + \log_2(3y+1)^4$

$\log_2 5 + 4\log_2(3y+1)$

11. $\log_9 \frac{1}{3} + 4\log_9 3$

$$\log_9\left(\frac{1}{3} \cdot 3^4\right)$$

$$\log_9\left(\frac{81}{3}\right) = \boxed{\log_9(27)}$$

12. $\log_{16}(\frac14) + 3\log_{16}4$

$$\log_{16}\left(\frac{1}{4} \cdot 4^3\right)$$
$$= \log(4^2)$$
$$= \boxed{\log(16)}$$

13. $4\log2 + \log7 - \log4$

$$\log\left(\frac{2^4 \cdot 7}{4}\right) = \log\left(\frac{16 \cdot 7}{4}\right)$$
$$= \boxed{\log(28)}$$

14. $\ln8 - 3*\ln2$

$$\ln\left(\frac{8}{2^3}\right) = \ln\left(\frac{8}{8}\right)$$
$$= \boxed{\ln(1)}$$

✏ **Practice:** Write an equivalent expression to expand each logarithm.

15. $\log_4(3\sqrt{x})$

$$\log_4 3 + \tfrac{1}{2}\log_4 x$$

16. $\log_3(8*(3x-1)^2)$

$$\log_3(8) + 2\log_3(3x-1)$$

17. $\log_8(2*(4y+5)^4)$

$$\log_8 2 + 4\log_8(4y+5)$$

18. $\log_{11}(4\sqrt{5})$

$$\log_{11}(4) + \tfrac{1}{2}\log_{11}(5)$$

19. $\log_4\sqrt{\dfrac{3w}{z}}$

$$\log_4(3w)^{\frac12} - \log_4 z^{\frac12}$$
$$\tfrac{1}{2}(\log_4(3w) - \log_4 z)$$
$$\tfrac{1}{2}(\log_4(3) + \log_4(w) - \log_4 z)$$

20. $\log_8\sqrt{\dfrac{s}{6r}}$

$$(\log_8(s) - (\log_8(6) + \log_8(r)))^{\frac12}$$
$$\tfrac{1}{2}(\log_8(s) - (\log_8(6) + \log_8(r)))$$

(A) $8\log_3\left(\sqrt{\dfrac{8x}{y}}\right)$

(B) $6\ln\left(\sqrt[3]{2x}\right)$

(C) $12\ln\left(\sqrt[4]{\dfrac{mn}{p}}\right)$

(D) $\log\left(\dfrac{7m^3 n^2}{q^4}\right)$

(E) $\log_3\left(7x\sqrt{y}\right)$

(F) $\ln\left(5(3y+4)^7\right)$

Exit Slip   Expand each. :)

(A) $4(\log_3 8 + \log_3 X - \log_3 Y)$

(B) $2(\ln 2 + \ln X)$

(C) $3(\ln(m) + \ln(\omega) - \ln(p))$

(D) $\log 7 + 3\log m + 2\log N - 4\log q$

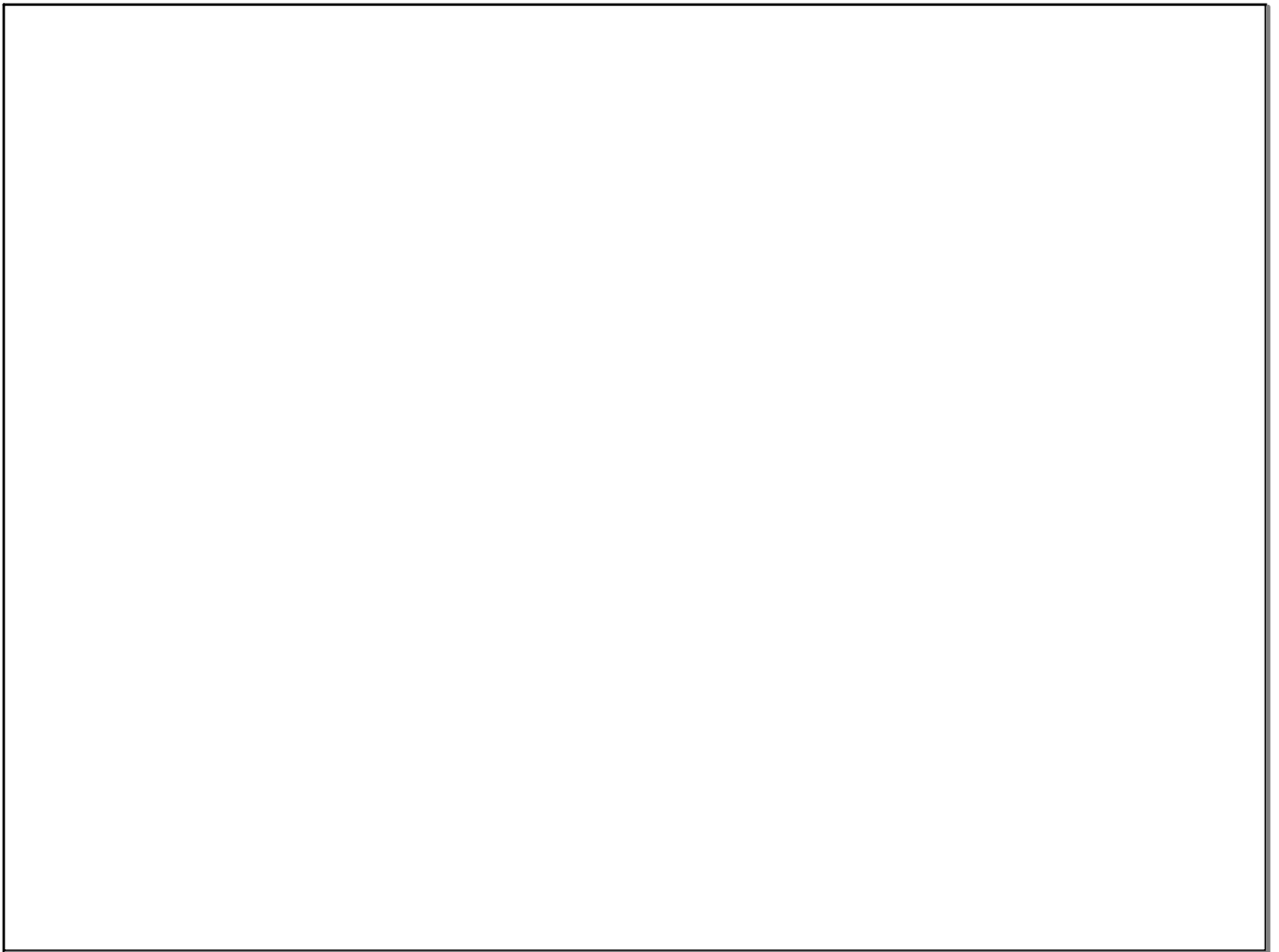(E) $\log_3 7 + \log_3 X + \frac{1}{2}\log_3 Y$

(F) $\ln 5 + 7\ln(3y + 4)$

Exit Slip   Expand each. ☺

1. $\log_3 \left(2(x+1)^3 \right)$

2. $\ln \left(4r \sqrt{S} \right)$

3. $\log_8 \left(\frac{c}{d^2} \right)$

4. $2\log_7 \left(\sqrt{\frac{5M}{N}} \right)$

Exit Slip: Write an equivalent expression to condense each logarithm.

1. $\log_2 2x + \log_2 x - \log_2 8$

2. $4\log 3x - \log 9$

Write an equivalent expression to expand each logarithm.

3. $\log_5(3^*(2x + 1)^2)$

4. $\ln(4x * y^2)$

<u>Homework:</u> pages 449-451

❦ <u>Condensing</u>: #11-18, 33, 37, 38, 41, 58, 60, 69, 73, 74, 77, 92, 93, and the extra problem below...

Extra Problem (Required): Condense $\log(9h) - 3(2\log(f) + \log(3h))$

❦ <u>Expanding</u>: #19-29 odds only, 43, 71, 79, 81, and the extra problem below...

Extra Problem (Required): Expand $\log\left(y^3\sqrt{\dfrac{w}{4z}}\right)$

❦ <u>Throwback</u>: p. 444 #100, 103, 104, and 105