$$2^x = 8$$

$$3^? = \frac{1}{27}$$

$$x = 3$$

$$-3$$

$$1000^{.05x} = 7500$$

Apr 18-8:17 AM

Introduction to Logarithms Class Work

✍ **Objective:** *You will be able to convert between logarithmic & exponential form.*

★ **FORMAL DEFINITION:**

Logarithm – a quantity representing the power to which a number (called the base) must be raised to produce a given number (called the argument)

*Logarithms are the "opposite" or "inverse" of exponentials.

★ **EXPONENTIAL FORM VS. LOGARITHMIC FORM**

$$\text{base}^{\text{exponent}} = \text{argument} \iff \log_{\text{base}}(\text{argument}) = \text{exponent}$$

ex. $3^{-3} = \frac{1}{27}$ ← argument $\iff \log_3\left(\frac{1}{27}\right) = -3$

base → exponent

$3^{-3} = \frac{1}{27}$ ✓

Examples (Converting Between Forms)

A. $2^3 = 8 \iff \log_2(8) = 3$

B. $3^2 = 9 \iff \log_3(9) = 2$

C. $\log_2(\frac{1}{4}) = \iff 2^{-2} = \frac{1}{4}$

D. $\log_5(125) = 3 \iff 5^3 = 125$

E. $\log_{49}7 = \iff 49^{\frac{1}{2}} = 7$

F. $x^0 = 1 \iff \log_x(1) = 0$

Examples (Evaluating Logarithms)

G. Evaluate $\log_7(343)$. $= x$ $7^x = 343$ ③

H. Evaluate $\log_6(1/36)$. $6^? = \frac{1}{36}$ ⊝-2

Examples (Evaluating Logarithms Part 2)

I. Evaluate $\log_8(16)$.

common base

$$8^x = 16$$
$$(2^3)^x = (2)^4$$
$$3x = 4$$
$$x = \boxed{4/3}$$

J. $\log_{128}(64)$.

$$128^x = 64$$
$$(2^7)^x = (2^6)$$
$$7x = 6$$
$$\boxed{x = 6/7}$$

1. $4^{-5} = \frac{1}{1024}$

$\log_4(1/1024) = -5$

2. $\log_{12}1728 = 3$

$12^3 = 1728$

3. $\log_{20}400 = 2$

$20^2 = 400$

4. $8^3 = 512$

$\log_8(512) = 3$

5. $\log_m n = p$

$m^p = n$

6. $w^v = z$

$\log_w(z) = v$

7. $\log_{-7}1 = 0$

$-7^0 = 1$

8. $9^{-1/2} = 1/3$

$\log_9(1/3) = -1/2$

✎ **Practice:** **(Evaluating Logarithms)** Evaluate each logarithm.

9. $\log_3 \frac{1}{27} =$ $3^x = \frac{1}{27}$ $\boxed{-3}$

10. $\log_8 16 =$

11. $\log_4 \frac{1}{64} =$ $4^x = 1/64$ $x = \boxed{-3}$

12. $\log_{10}10000 =$ $\boxed{4}$

13. $\log_3 1 =$ $3^x = 1$ $x = \boxed{0}$

14. $\log_{11}121 =$ $\boxed{2}$

15. $\log_4 2 =$ $4^x = 2$ $x = \boxed{1/2}$

16. $\log_8 \frac{1}{2} =$ $8^x = \frac{1}{2} \to (2^3)^x = 2^{-1}$ $3x = -1$ $\boxed{x = -1/3}$

17. $\log_8 8 =$

18. $\log_{1/3}3 =$ $\boxed{-1}$ $\frac{1}{3}^x = 3$

19. $\log_{512}8 =$ $512^x = 8$ $\boxed{x = 1/3}$ $8^{3x} = 8^1$

20. $\log_4 64 =$ $\boxed{-3}$ $\left(\frac{1}{4}\right)^x = 64$

***What about $\log_6(-36)$?!? $6^x = -36$ (No real solution argument cannot be negative ∴)

★ **Most common Logarithm**

✧ A common logarithm is **log base 10**. The base of 10 is typically not written.

Practice:

21. Evaluate log(100) $10^x = 100$ ②

22. Evaluate log (1/1000)

23. If log(x) = -2, what is x? $10^{-2} = x$ $\boxed{1/100}$

24. If log(y) = 4, what is y?

✎ **Practice:** **(Evaluating Logarithms Part 2)** Evaluate each logarithm.

25. $\log_{125}\frac{1}{25}$ $125^x = 1/25$ $(5^3)^x = 5^{-2}$ $3x = -2$ $\boxed{x = -2/3}$

26. $\log_4 \frac{1}{16} = -2$ $(4)^x = (4^{-2})$ $\boxed{x = -2}$ $\boxed{x = -2}$

27. $\log_3 (27) = 3$ $81^x = 27_3$ $4x = 3$ $\boxed{x = 3/4}$

28. $\log_{27}\frac{1}{729}$ $(3^3)^x = (3^{-6})$ $3x = -6$

29. $\log_{10}\frac{1}{10}$ $10^x = \frac{1}{10}$ $\boxed{-1}$

30. $\log_{216}36 =$ $216^x = 36$ $(6^3)^x = 6^2$ $\frac{3x}{3} = \frac{2}{3}$ $\boxed{x = 2/3}$

How has your prior knowledge been helpful in learning about logarithms?!
What questions do you still have? ☺

22) $\log(1/1000)$

$10^x = \frac{1}{1000}$ (−3)

24) If $\log(y) = 4$ $y = ?$

$10^4 = y$ ($y = 10000$)

Exit Slip Converting: Complete on a separate sheet of paper please.

1. Write $3^{-2} = 1/9$ in logarithmic form.   2. Write $\log_2 32 = 5$ in exponential form.

Exit Slip Evaluating: Complete on a separate sheet of paper please.

3. Evaluate $\log_6(216)$.   4. Evaluate $\log_2(1/32)$.

✧ Another common logarithm is **log base e** (also known as the **natural log**). You can evaluate the natural log of a number using the "LN" button on your calculator! ☺

> ***e** is a mathematical constant, also known as Euler's number, which is helpful in solving exponential functions. The number is irrational and begins with 2.718281828459...

> *We will use the natural logarithm to our advantage pretty soon!

<u>Practice:</u> Evaluate each logarithm using your calculator. Round to the nearest ten-thousandth where necessary.

1. $\ln(8) =$ _**2.0794**_   2. $\log(9) =$ _**0.9542**_ $\log(0) =$ _**undefined**_ 4. $\ln(0.235) =$ _**-1.4482**_

5. $\ln(5.2/1.3) =$ _**1.3863**_ 6. $\log(-3) =$ _**undefined**_ $\log(10^{28}) =$ _**28**_ 8. $\ln(e^{2.3}) =$ _**2.3**_

$$10^x = 10^{28} \qquad e^x = e^{2.3}$$

<u>Exit Slip Converting</u>: Complete on a separate sheet of paper please.

1. Write $3^{-2} = 1/9$ in logarithmic form.   2. Write $\log_2 32 = 5$ in exponential form.

<u>Exit Slip Evaluating</u>: Complete on a separate sheet of paper please.

3. Evaluate $\log_6(216)$.   4. Evaluate $\log_2(1/32)$.

(red handwriting: include name)

<u>Exit Slip Common Log</u>: Complete on a separate sheet of paper please.

5. Evaluate $\log(10,000)$.   6. If $\log(1/10) = p$, what is p?   *(red: p = 1/10)*

<u>Exit Slip Evaluating Part 2</u>: Complete on a separate sheet of paper please.

7. Evaluate $\log_{343}(49)$.

Homework: pages 442-444

<u>Converting</u>: #6, 7, 9, 11, 12, 13, 53, 56-60

<u>Evaluating</u>: #14, 16, 18-25, 50, 93-96

<u>Evaluating Part 2</u>: #15, 17, 100 – 103 and 107-109 (throwback, required because practice for the final exam and to prepare for next year!

The following is also part of this section…

Evaluate:

A. $\log_{1/4}(8)$ B. $\log_{125}(1/5)$ C. $\log_{1/81}(27)$ D. $\log_{16}(1/32)$

*All solutions are on the Unit 7 HW page! ☺ Please ask questions if you have any!

Apr 19-8:50 AM