

Systems of Equations Pre-Unit Refresher

Please make sure to refresh your memory of these ideas prior to the first day of this unit. A strong conceptual understanding of these topics is absolutely essential for success!

Please try all practice problems that are provided in this packet, and check out the links to supplemental videos and extra practice problems. Be sure to let me know if you still need help understanding any of these mathematical concepts. After checking your answers, please retry any problems you had incorrect, and discuss any specific questions you have relating to any of the ideas/problems in this packet with me.

Remember, my goal this year (and my goal in providing this assignment) is to help you and prepare you, *not* to stress you! :))

****Supplemental Videos:***

- **Systems:** <https://www.khanacademy.org/math/algebra-basics/core-algebra-systems>
- **Graphing Lines:** <https://www.khanacademy.org/math/algebra-basics/core-algebra-graphing-lines-slope#core-algebra-graphing-slope-intercept>
- **Graphing Absolute Value Functions:** <https://www.khanacademy.org/math/algebra/absolute-value-equations-functions#graphs-of-absolute-value-functions>
- **Graphing Inequalities:** <https://www.khanacademy.org/math/algebra/two-variable-linear-inequalities>

****Online Resources and Practice Problems:***

- **Systems:** <http://www.regentsprep.org/regents/math/algebra/ae3/indexae3.htm>
- **Systems:** <https://www.ixl.com/math/algebra-1> (section U)
- **Graphing Lines:** <http://www.regentsprep.org/regents/math/algebra/ac1/eqlines2.htm>
- **Graphing Lines:** <https://www.ixl.com/math/algebra-1/slope-intercept-form-graph-an-equation>
- **Graphing Horizontal and Vertical Lines:** <https://www.ixl.com/math/algebra-1/graph-a-horizontal-or-vertical-line>
- **Graphing Absolute Value Functions:** <https://www.ixl.com/math/algebra-1/graph-an-absolute-value-function>
- **Graphing Inequalities:** <http://www.regentsprep.org/regents/math/algebra/ae85/indexAE85.htm>

A. Solving Systems of Equations (Graphing)

****Try These Practice Problems!***

1. What is a “solution” to a system of equations, and what does a “solution” to a system of equations look like on a graph of the equations?
2. If two lines are parallel, how many solutions exist, and why?
3. If two lines are overlapping, how many solutions exist, and why?

4. Determine how many solutions each system will have, and support your answer.

a. $y = -x + 1$
 $x = -y - 1$

b. $-4y = 4 + x$
 $\frac{1}{4}x + y = -1$

c. $3x + 6y = 18$
 $3y + 1.5x = 9$

d. $2y = 4x - 8$
 $10 - 3y = -6x$

e. $\frac{1}{2}x - 3 = y$
 $-2y = 4x + 3$

f. $5y - 10 = -7x$
 $10y + 14x = 20$

g. $y = |x + 2| + 1$
 $y = -|x + 2| + 1$

h. $y = |x|$
 $y = x$

5. Solve each system by graphing (use graph paper).

a. $2y = -3x - 4$
 $6x - 4y = 8$

b. $2x + y = 9$
 $-x - 2y = -3$

c. $2x + 4y = -20$
 $3x - 9y = 0$

d. $-2x - 4y = -10$
 $-x - 2y = -4$

e. $y = 9$
 $x = -8$

f. $\frac{1}{2}y = |x + 4|$
 $y = -|x| - 3$

g. $y = |x - 3|$
 $-\frac{1}{2}y = |x - 3|$

h. $y = |x + 1|$
 $y = -x - 1$

B. Solving Systems of Equations (Substitution)

****Try These Practice Problems!***

6. Sometimes when solving systems, you may encounter a true statement when solving for one variable (such as $0 = 0$, or $3 = 3$, or $-2 = -2$, etc.). How many solutions would such a system have?

7. Sometimes when solving systems, you may encounter a false statement when solving for one variable (such as $8 = 0$, or $7 = 3$, or $-1 = -2$, etc.). How many solutions would such a system have?

8. Solve each system using the substitution method.

a. $-2x + 8y = -14$
 $24y - 6x = -42$

b. $-9x - 4y = -4$
 $-6y - 18x = 12$

c. $2y = -x + 1$
 $4.5x - 9 = 9y$

d. $2x + 3y = 5$
 $x = 10$

*What about the system consisting of $y = x^2$ and $y = 2x$??

C. Solving Systems of Equations (Elimination)

****Try These Practice Problems!***

9. Solve each system using the elimination method.

- | | | | |
|--------------------|----------------------|---------------------|------------------|
| a. $3x - 12y = 30$ | b. $2x - 4y + 8 = 0$ | c. $45x + 54y = 18$ | d. $-3y = x - 1$ |
| $-6x + 15y = -15$ | $12x - 4y = 24$ | $-36y - 90x = 72$ | $2x + 4y = 0$ |

D. Miscellaneous Essential Skills

10. For each situation, define two variables in words. Use any variables of your choice for the unknown values. Then write an equation to represent each situation.

- a. Olivia baked a total of 200 cookies to bring to a homeless shelter. Some were chocolate chip, and the rest were sugar cookies.
- b. Five cucumbers and three tomatoes cost a total of \$6.70.
- c. The perimeter of a field is 400 feet.

11. One of Mike's Babe Ruth baseball cards is valued at b dollars, and another card he has is worth 30% more than the Babe Ruth card. Write an expression for the value of the second baseball card in terms of the value of the Babe Ruth card.

12. Multiple choice: Which interval does the value $x = -2.3$ fall within?

- | | | | |
|------------------|------------------|-----------------|----------------|
| a. $-4 < x < -3$ | b. $-3 < x < -2$ | c. $-2 < x < 2$ | d. $2 < x < 4$ |
|------------------|------------------|-----------------|----------------|

13. Multiple choice: Which interval does the value $x = 1.89$ fall within?

- | | | | |
|-----------------|------------------|------------------|------------|
| a. $-2 < x < 0$ | b. $0 < x < 1.8$ | c. $1.8 < x < 2$ | d. $x > 2$ |
|-----------------|------------------|------------------|------------|

E. Graphing Inequalities

For problems 1 through 8, graph each equation or inequality.

*Use a test point (like the origin, (0,0)) to decide where to shade. If you end up with a TRUE inequality, shade the area INCLUDING the point. If you end up with a FALSE inequality, shade the area OUTSIDE of the point.

For example, if you test (0,0) in problem one, you would replace x with 0 and y with zero, resulting in

$$0 < -|0 - 4| + 3?$$

$$0 < -|-4| + 3?$$

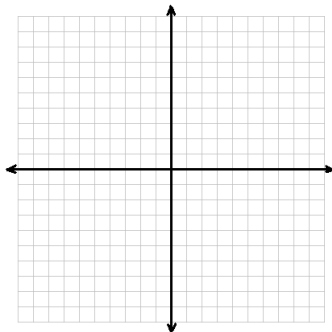
$$0 < -4 + 3?$$

$$0 < -1?$$

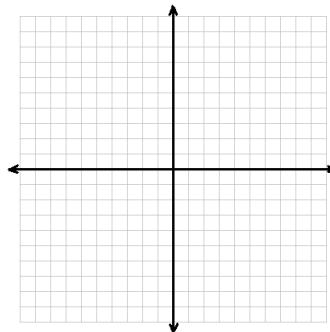
FALSE, so DO NOT include (0,0) when shading! 😊

*Also remember to use a dashed line for $<$ or $>$ and a solid line for \leq and \geq .

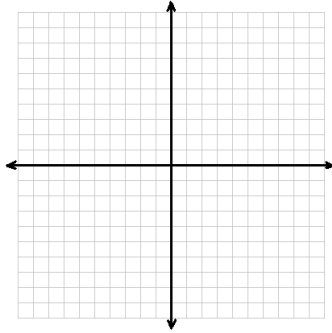
1. $y < -|x - 4| + 3$



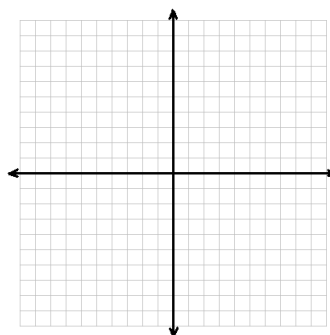
2. $2x + 3y > 12$



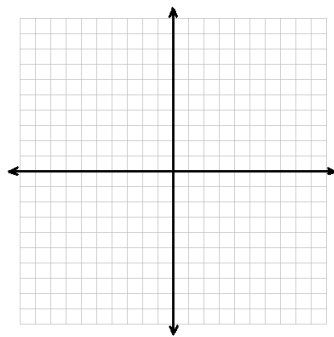
3. $3x - 2y \geq 4$



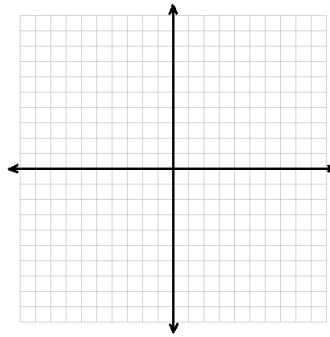
4. $y < |x + 3|$



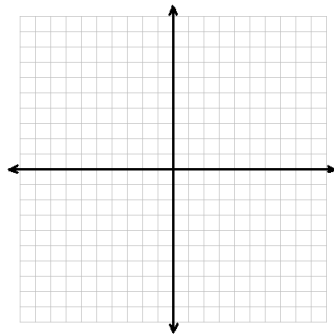
5. $f(x) \leq |x - 8| + 1$



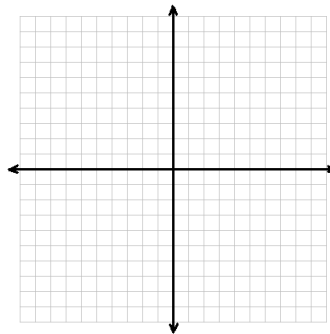
6. $-x + 2y < 8$



6. $y > -|x| + 9$



8. $4y - 8x < -24$



Solutions:

A. Solving Systems of Equations (Graphing)

1. The solution to a system of equations is the point on the graph where the graphs of the equations intersect. The x-values produce the same y-values in both equations at this point.
2. If two lines are parallel, the system of equations for the lines has no solutions, since the lines will never intersect. There is no x-value that will produce the same y-value in both equations, since the slopes are the same and the y-intercepts are different.
3. If two lines are overlapping, the system of equations for the lines has infinite solutions, since the lines will intersect at all points. Every x-value will produce the same y-value in both equations, since the slopes are the same and the y-intercepts are the same.
- 4.

4) a. $y = -x + 1$
 $x = y - 1 \rightarrow y + x = -1$
 $y = -x - 1$
same slope, different y-intercepts \rightarrow parallel
* thus, no solutions.

b. $-4y = 4 + x$
 $y = -\frac{1}{4}x - 1$
 $\frac{1}{4}x + y = -1$
 $y = -\frac{1}{4}x - 1$
 \rightarrow same equation \rightarrow same line
* thus, infinitely many solutions.

c. $3x + 6y = 18$
 $6y = -3x + 18$
 $y = -\frac{1}{2}x + 3$
 $3y + 1.5x = 9$
 $3y = -1.5x + 9$
 $y = -\frac{1}{2}x + 3$
 \rightarrow same equation \rightarrow infinitely many solutions

d. $2y = 4x - 8$
 $y = 2x - 4$
 $10 - 3y = -6x$
 $-3y = -6x - 10$
 $y = 2x + \frac{10}{3}$
 \rightarrow same slope only \rightarrow parallel
 \rightarrow no solutions

e. $\frac{1}{2}x - 3 = y$
 $y = \frac{1}{2}x - 3$
 $-2y = 4x + 3$
 $y = -2x - \frac{3}{2}$
 \rightarrow completely different equations (actually perpendicular)
one solution

f. $5y - 10 = -7x$
 $5y = -7x + 10$
 $y = -\frac{7}{5}x + 2$
 $10y + 14x = 20$
 $10y = -14x + 20$
 $y = -\frac{7}{5}x + 2$
 \rightarrow same line \rightarrow infinitely many solutions

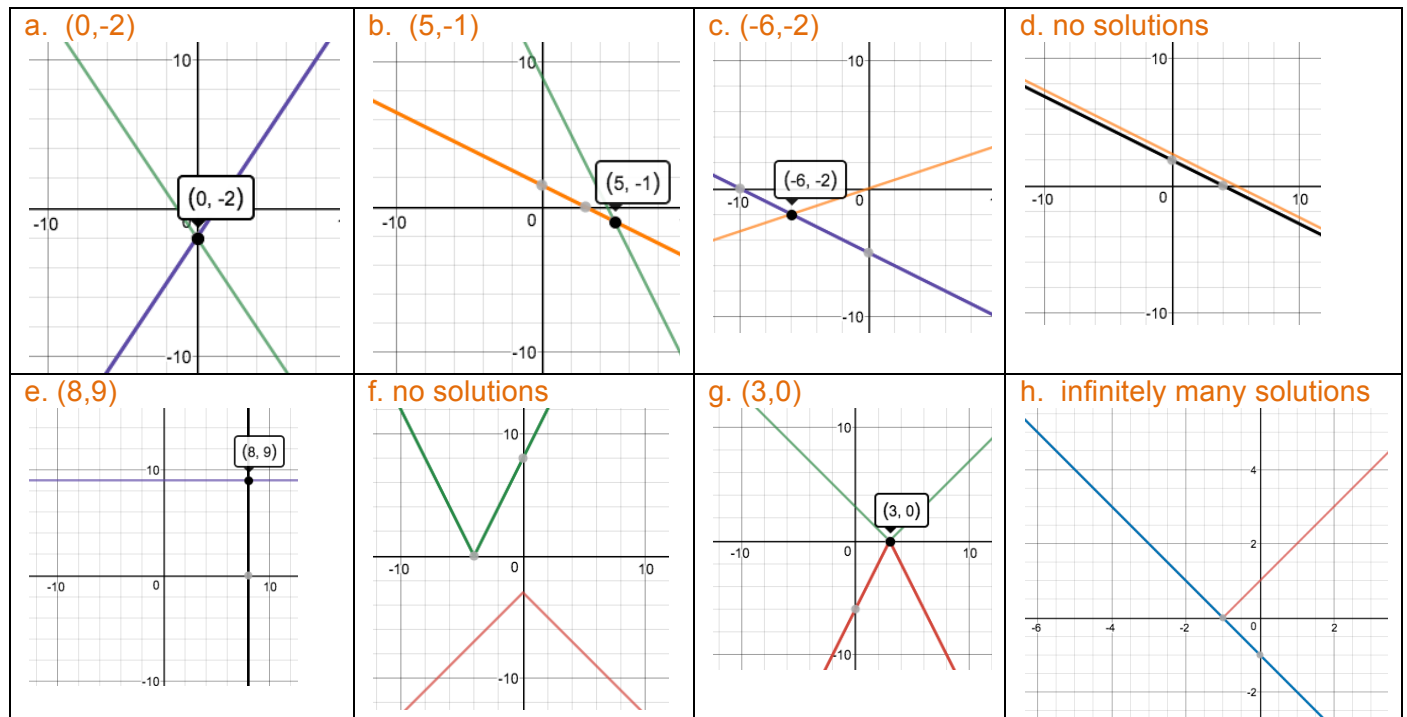
g. $y = |x + 2| + 1$
vertex: $(-2, 1)$
opens up
 $y = -|x + 2| + 1$
vertex: $(-2, 1)$
opens down

1 solution (the vertex)

h. $y = |x|$
vertex $(0, 0)$
opens up
 $y = x$
 $m = 1$
 $y\text{-int: } (0, 0)$

 \rightarrow overlap, infinitely many solutions

5.



B. Solving Systems of Equations (Substitution)

6. The system would have infinite solutions, since the resulting statement is true.

7. The system would have zero solutions, since the resulting statement is false.

8.

$$8) \begin{cases} -2x + 8y = -14 \\ -6x + 24y = -42 \end{cases}$$

$$\rightarrow \begin{cases} -2x = -8y - 14 \\ x = 4y + 7 \end{cases}$$

$$\rightarrow \begin{aligned} -6(4y + 7) + 24y &= -42 \\ -24y - 42 + 24y &= -42 \\ -42 &= -42 \end{aligned}$$

True, so infinite solutions

$$b) \begin{cases} -9x - 4y = -4 \\ -6y - 18x = 12 \end{cases}$$

$$\rightarrow \begin{cases} -6y = 18x + 12 \\ y = -3x - 2 \end{cases}$$

$$\rightarrow \begin{aligned} -9x - 4(-3x - 2) &= -4 \\ -9x + 12x + 8 &= -4 \\ 3x + 8 &= -4 \\ 3x &= -12 \\ x &= -4 \end{aligned}$$

$$\rightarrow \begin{aligned} y &= -3(-4) - 2 \\ y &= 12 - 2 \\ y &= 10 \end{aligned}$$

so $(-4, 10)$

$$c) \begin{cases} 2y = -x + 1 \\ 4.5x - 9 = 9y \end{cases} \rightarrow \begin{cases} y = -\frac{1}{2}x + \frac{1}{2} \end{cases}$$

$$\rightarrow 4.5x - 9 = 9(-\frac{1}{2}x + \frac{1}{2})$$

$$4.5x - 9 = -4.5x + 4.5$$

$$9x = 13.5$$

$$x = 1.5$$

$$\rightarrow \begin{aligned} y &= -\frac{1}{2}(1.5) + \frac{1}{2} \\ y &= -.25 \end{aligned}$$

so $(1.5, -.25)$

$$d) \begin{cases} 2x + 3y = 5 \\ x = 10 \end{cases} \rightarrow \begin{aligned} 2(10) + 3y &= 5 \\ 20 + 3y &= 5 \end{aligned}$$

$$3y = -15$$

$$y = -5$$

so $(10, -5)$

*What about the system consisting of $y = x^2$ and $y = 2x$??

$$x^2 = 2x$$

$$x^2 - 2x = 0$$

$$x(x - 2) = 0$$

so $x = 0$ and $x - 2 = 0$,

resulting in $x = 0$, and $x = 2$

When $x = 0$, $y = 0^2$ and $y = 2(0)$, so $y = 0$

One solution is the point $(0,0)$

When $x = 2$, $y = 2^2$ and $y = 2(2)$, so $y = 4$

The other solution is the point $(2, 4)$.

C. Solving Systems of Equations (Elimination)

9.

9) a. $3x - 12y = 30$ (2)
 $-6x + 15y = -15$

$$\begin{array}{r} 6x - 24y = 60 \\ -6x + 15y = -15 \\ \hline 0x - 9y = 45 \\ y = -5 \end{array}$$

$$\begin{array}{r} 3x - 12(-5) = 30 \\ 3x + 60 = 30 \\ 3x = -30 \\ x = -10 \end{array}$$

$$\boxed{(-10, -5)}$$

b. $2x - 4y + 8 = 0$ (-1)
 $12x - 4y = 24$

$$\begin{array}{r} -2x + 4y - 8 = 0 \\ 12x - 4y = 24 \\ \hline 10x = 32 \\ x = 3.2 \end{array}$$

$$\begin{array}{r} -2x + 4y = 8 \\ 12x - 4y = 24 \\ \hline 10x = 32 \\ x = 3.2 \end{array}$$

$$\begin{array}{r} 12(3.2) - 4y = 24 \\ 38.4 - 4y = 24 \\ -4y = -14.4 \\ y = 3.6 \end{array}$$

$$\boxed{(3.2, 3.6)}$$

rearrange to...

c. $45x + 54y = 18$ (2)
 $-90x - 36y = 72$

$$\begin{array}{r} 90x + 108y = 36 \\ -90x - 36y = 72 \\ \hline 0x + 72y = 108 \\ y = \frac{3}{2} \end{array}$$

$$\begin{array}{r} 45x + 54(\frac{3}{2}) = 18 \\ 45x + 81 = 18 \\ 45x = -63 \\ x = -\frac{7}{5} \text{ or } -1.4 \end{array}$$

$$\boxed{(-1.4, 1.5)}$$

rearrange to...

d. $-x - 3y = -1$ (2)
 $2x + 4y = 0$

$$\begin{array}{r} -2x - 6y = -2 \\ 2x + 4y = 0 \\ \hline 0x - 2y = -2 \\ y = 1 \end{array}$$

$$\begin{array}{r} -x - 3(1) = -1 \\ -x - 3 = -1 \\ -x = 2 \\ x = -2 \end{array}$$

$$\boxed{(-2, 1)}$$

D. Miscellaneous Essential Skills

10. Write an equation to represent each situation. Use any variables of your choice for the unknown values.

a. c: number of chocolate chip cookies; s: number of sugar cookies

equation: $c + s = 200$

b. c: the price of one cucumber, in dollars; t: the price of one tomato, in dollars

$$5c + 3t = 6.70$$

c. L: length of the field, in feet; W: width of the field, in feet

$$2(L + W) = 400$$

11. $1.30b$ (100% of the first card + 30% of the first card = 130% of the first card; 130% is 1.30 in decimal form)

12. Multiple choice: Which interval does the value $x = -2.3$ fall within?

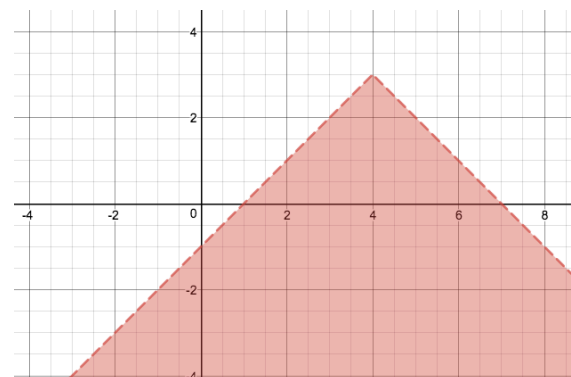
b. $-3 < x < -2$

13. Multiple choice: Which interval does the value $x = 1.89$ fall within?

c. $1.8 < x < 2$

E. Linear Equalities

1. $y < -|x - 4| + 3$

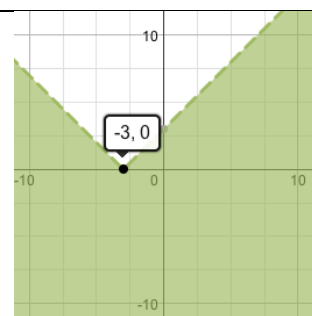
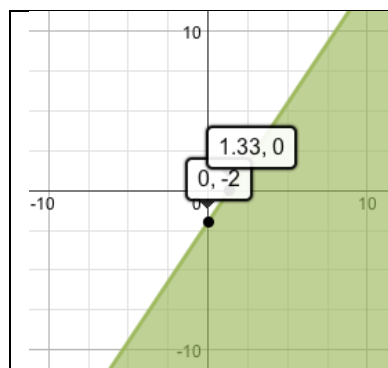


2. $2x + 3y > 12$

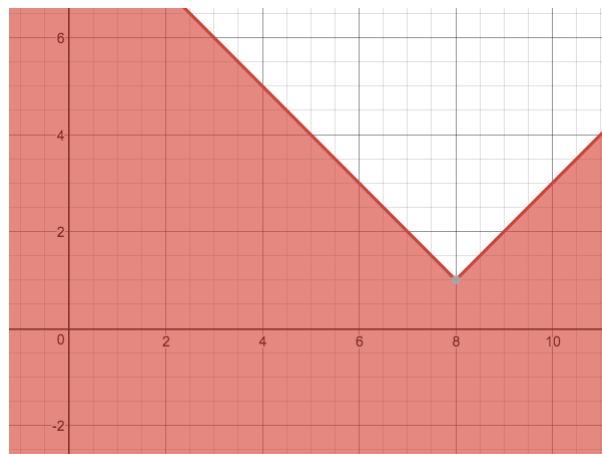


3. $3x - 2y \geq 4$

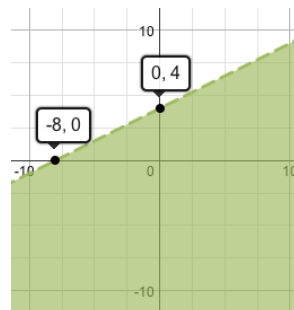
4. $y < |x + 3|$



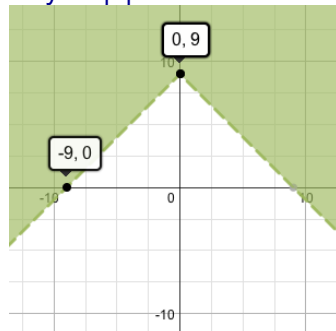
5. $f(x) \leq |x - 8| + 1$



6. $-x + 2y < 8$



6. $y > -|x| + 9$



8. $4y - 8x < -24$

